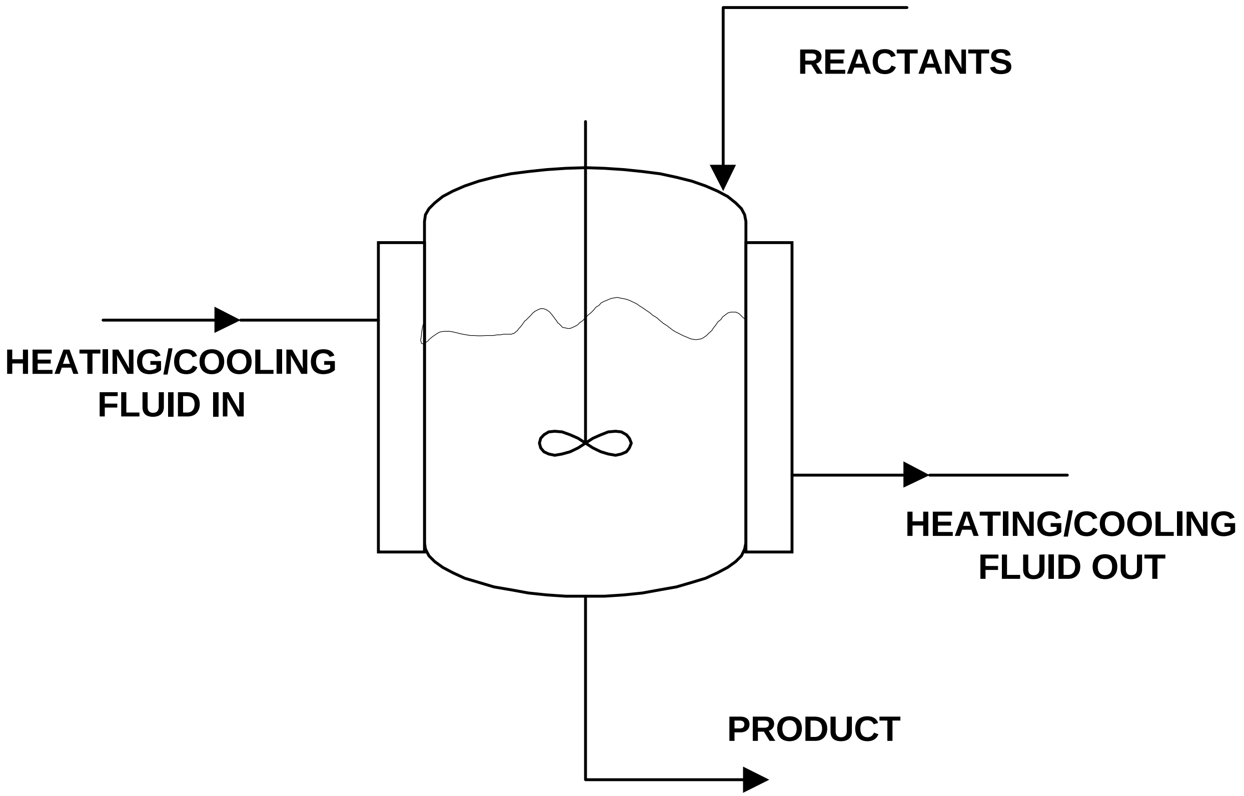
MULTILOOP SYSTEMS (Relative Gains)

Ming Tham

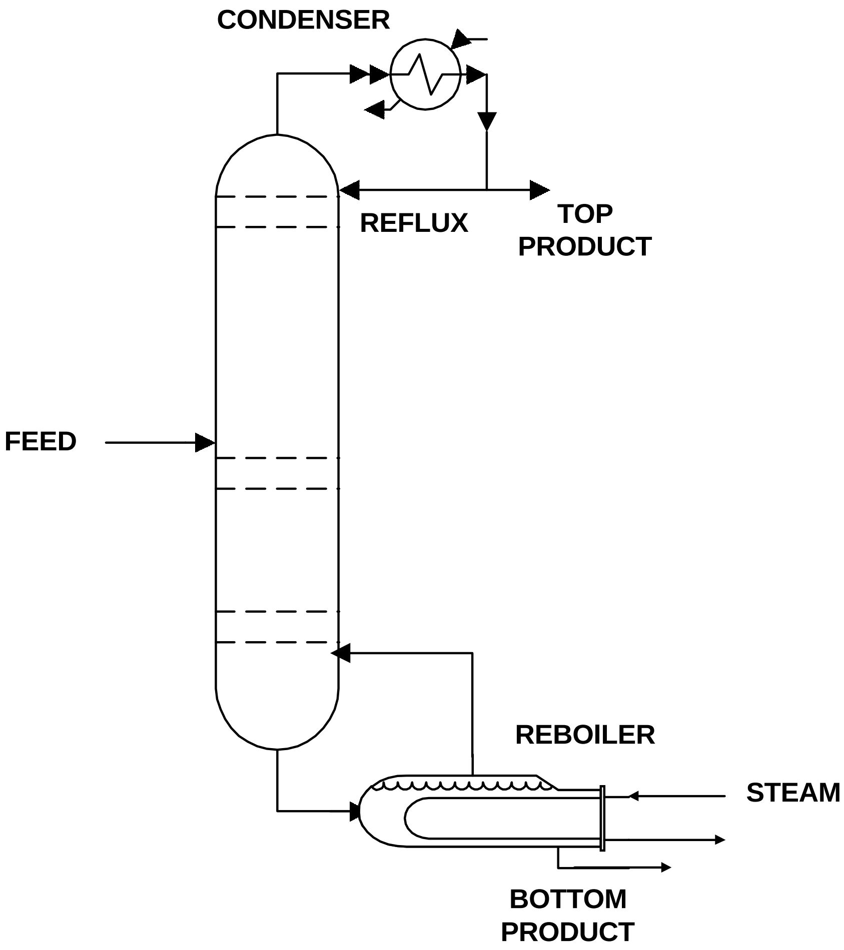
Scope

- Loop interactions
- Model structures
- Relative gains
- Selection of control loop pairings
- Summary and discussion

Example 1

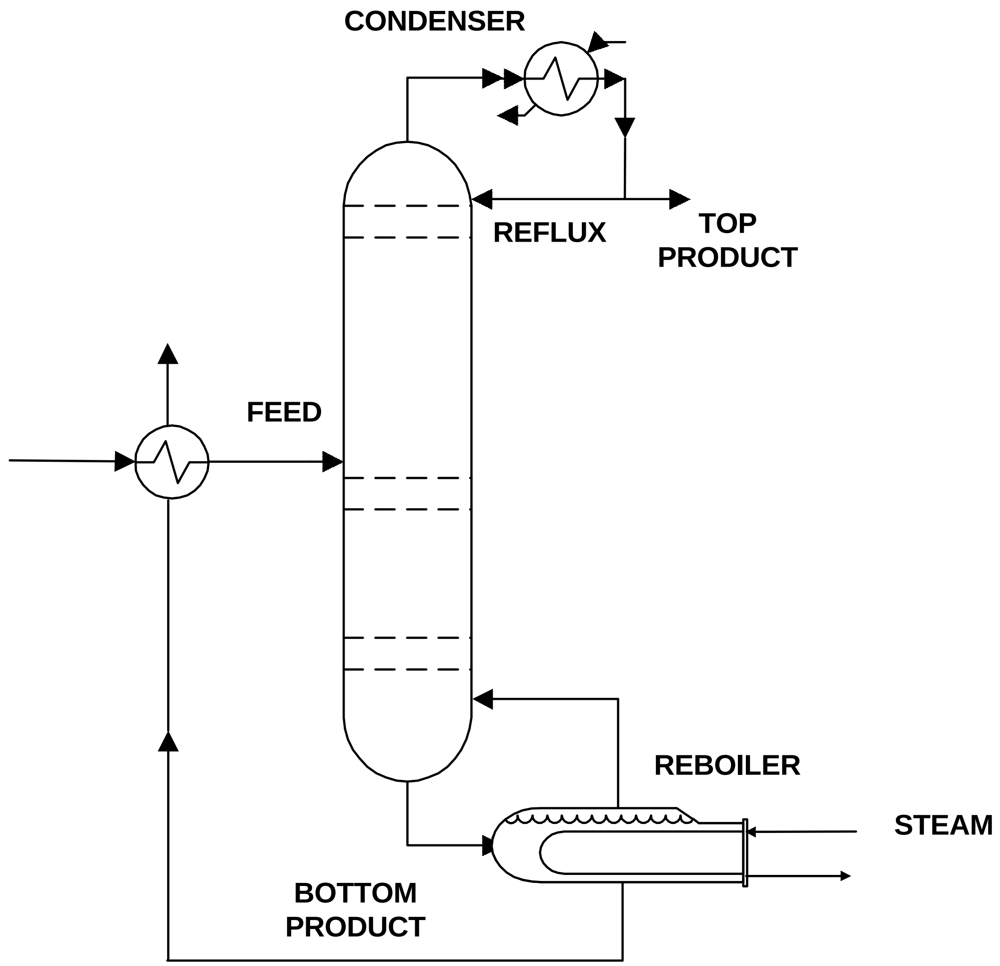

REACTANTS
HEATING/COOLING
FLUID IN
HEATING/COOLING
FLUID OUT
PRODUCT

Example 2



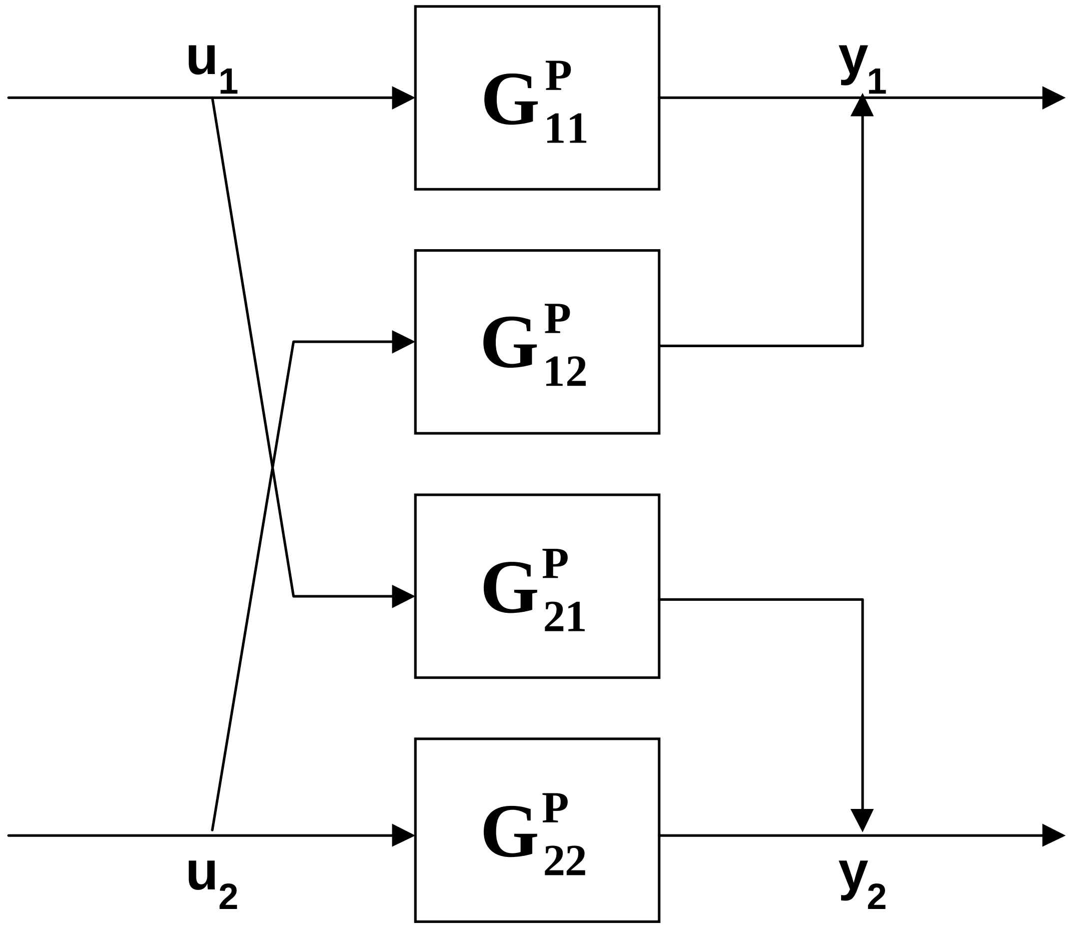
CONDENSER
REFLUX
TOP PRODUCT
FEED
REBOILER
STEAM
BOTTOM PRODUCT

Example 3

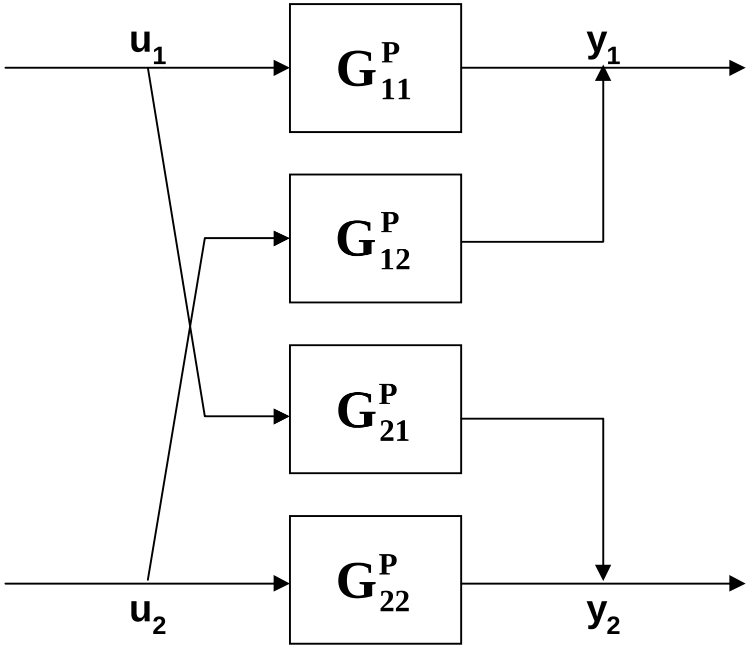


CONDENSER
REFLUX
TOP PRODUCT
FEED
REBOILER
STEAM
BOTTOM PRODUCT

P-canonical structure



P-canonical structure



$$y_1 = u_1 G_{11}^p + u_2 G_{12}^p$$

$$y_2 = u_1 G_{21}^p + u_2 G_{22}^p$$

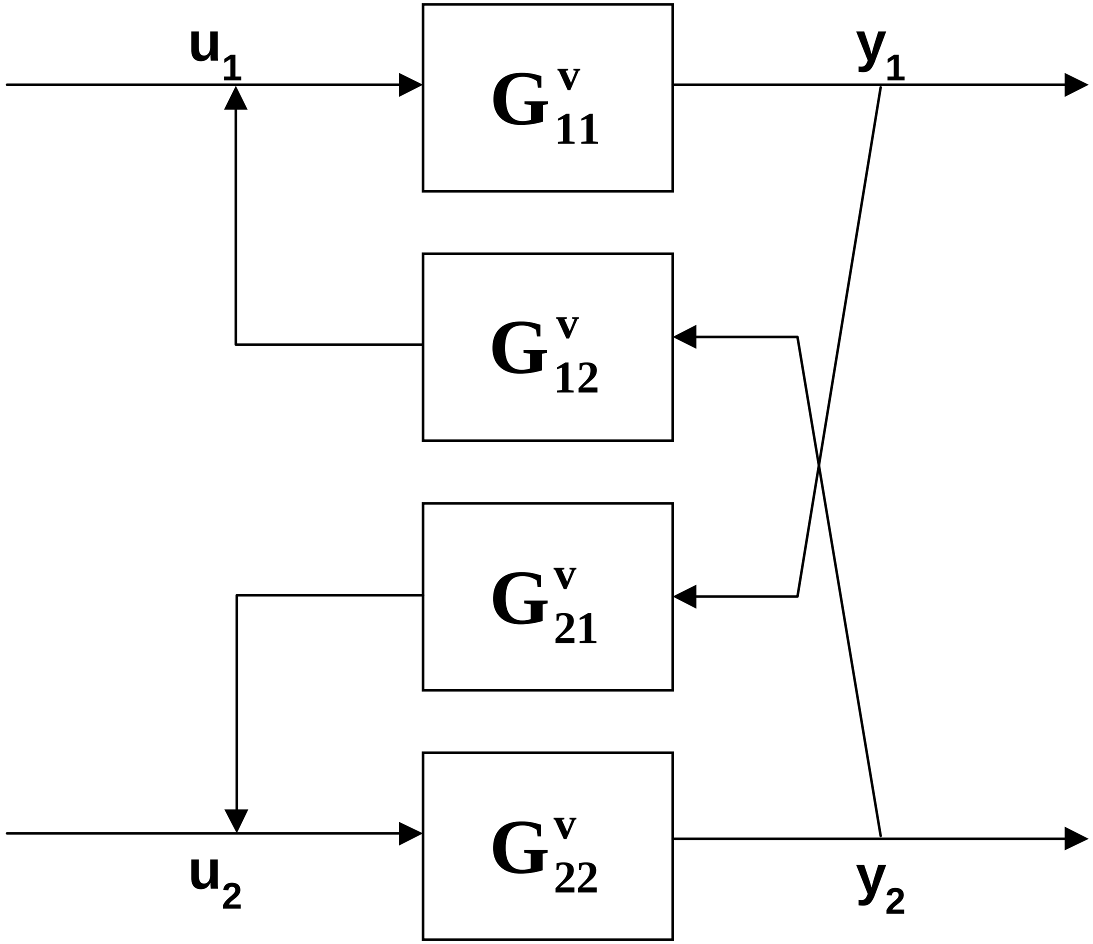
$$\mathbf{y} = \mathbf{G^p u}$$

$$\mathbf{y} = [y_1, y_2]^T$$

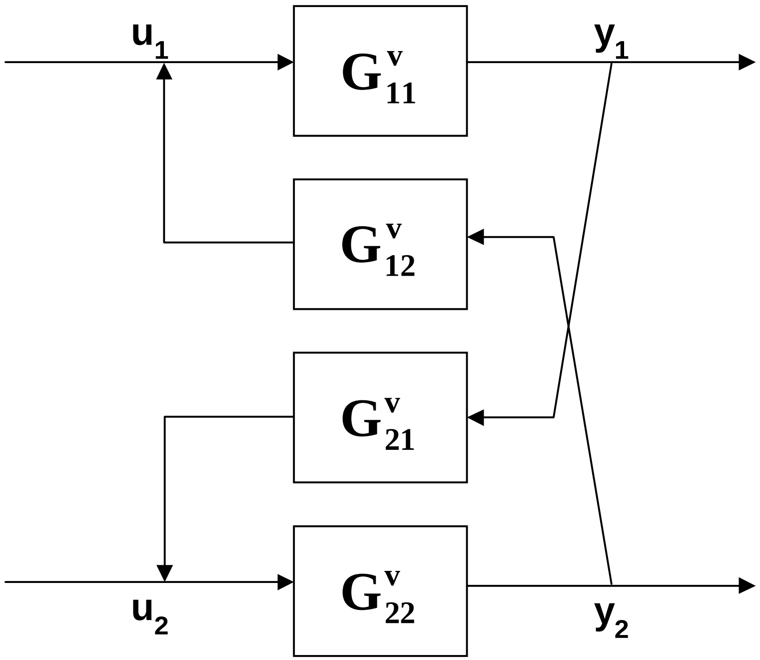
$$\mathbf{u} = [u_1, u_2]^T$$

$$\mathbf{G^p} = \begin{bmatrix} G_{11}^p & G_{12}^p \\ G_{21}^p & G_{22}^p \end{bmatrix}$$

V-canonical structure



V-canonical structure



$$y_1 = \left[y_2 G_{12}^v + u_1 \right] G_{11}^v$$

$$y_2 = \left[y_1 G_{21}^v + u_2 \right] G_{22}^v$$

$$\mathbf{y} = \left[\mathbf{I} - \mathbf{G_m^v} \mathbf{G_i^v} \right]^{-1} \mathbf{G_m^v} \mathbf{u}$$

$$\mathbf{G_m^v} = \begin{bmatrix} G_{11}^v & 0 \\ 0 & G_{22}^v \end{bmatrix}$$

$$\mathbf{G_i^v} = \begin{bmatrix} 0 & G_{12}^v \\ G_{21}^v & 0 \end{bmatrix}$$

P- and V-canonical structures

- If both P- and V- structures are used to represent the same process,

$$\mathbf{y} = \mathbf{G}^{\mathbf{p}}\mathbf{u}$$

$$\mathbf{y} = \left[\mathbf{I} - \mathbf{G}_{\mathbf{m}}^{\mathbf{v}}\mathbf{G}_{\mathbf{i}}^{\mathbf{v}}\right]^{-1}\mathbf{G}_{\mathbf{m}}^{\mathbf{v}}\mathbf{u}$$

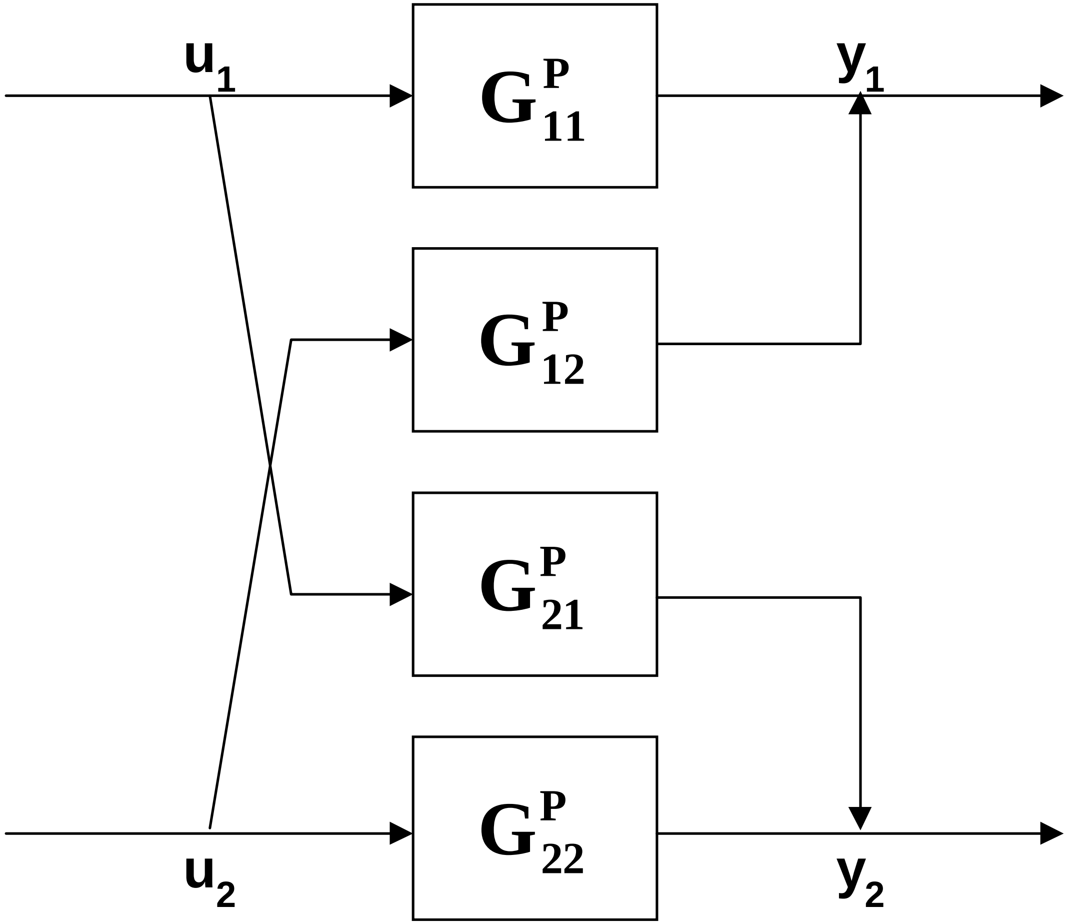
then the two structures are related according to:

$$\mathbf{G}^{\mathbf{p}} = \left[\mathbf{I} - \mathbf{G}_{\mathbf{m}}^{\mathbf{v}}\mathbf{G}_{\mathbf{i}}^{\mathbf{v}}\right]^{-1}\mathbf{G}_{\mathbf{m}}^{\mathbf{v}}$$

Choosing a model structure

- it should be possible to determine the parameters of the model from experiments
- the model must be representative of the process, and preferably general enough to encompass other processes
- the model should be able to provide the relevant information for control systems design
- the model should be simple

Which structure is better?



The Problem with loop interactions

- two pure first order systems under proportional control
- loops are non-interacting
- characteristic equations

$$1 + K_{p,1} G_{11} = 0 \qquad\qquad 1 + K_{p,2} G_{22} = 0$$

- both loops are stable regardless of gains of controllers

The problem with loop interactions

- 2x2 interacting system
- proportional control on each output
- characteristic equation is

$$\left(1+K_{p,1}G_{11}\right)\left(1+K_{p,2}G_{22}\right)-G_{12}K_{p,2}G_{21}K_{p,1}=0$$

- there is a range of gains beyond which the system will become unstable, even though the process transfer functions are pure first order systems (-ve sign in characteristic equation)

Statement of the interaction problem

"If the steady state or dynamic gain of a given controlled variable in response to a given manipulated variable changes when other (initially open) loops are closed, then interaction exists in the system."

Statement of the interaction problem

"If the controller in question was tuned with all others in manual, that tuning will be incorrect when all the others are placed in automatic because of their influence over the gain of that particular control loop."

Statement of the interaction problem

"Depending on the degree of interaction, instability or at the very least, degraded responses will result."

Dealing with loop interactions

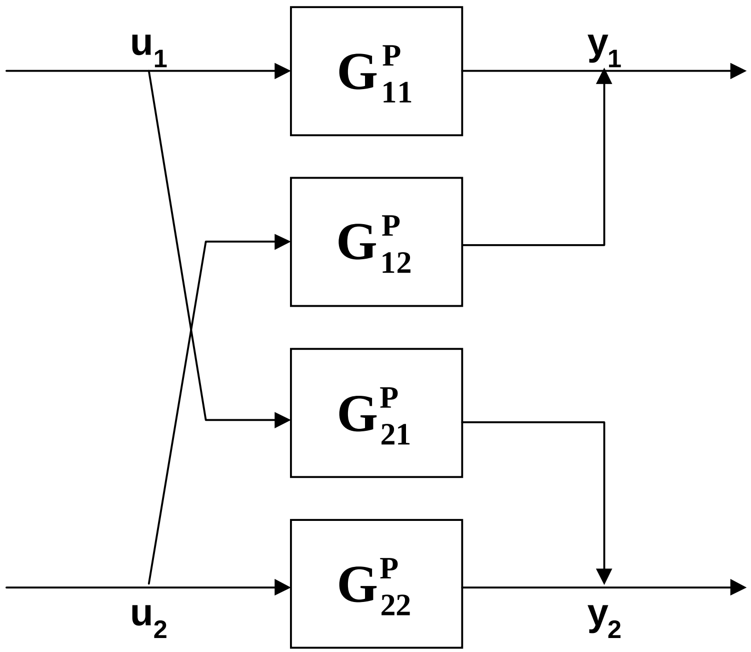
- Two approaches
  - given outputs that have to be controlled, choose manipulated inputs that will lead to the least interaction
  - design multivariable control algorithms or strategies that will eliminate or attenuate the effects of loop interactions
- Consider only control loop selection

Choosing suitable input-output pairs

- to be able to choose input-output pairs that yield the least interaction
  - some measure must be used to quantify the degree of interaction
  - the measure must be easy to determine
  - the measure must be applicable to a wide range of systems
  - this measure should also provide an insight into process behaviour
- one of the earliest interaction measure is the *Relative Gain*

Relative Gains

- ratio of process gains when the all loops are opened to the gains when all loops are closed



The relative gain for loop 1 is given by:

$$\lambda_{11} = K_{11}|_{u_2} \Big/ K_{11}|_{y_2}$$

$$K_{11}|_{u_2} = \left.\frac{\Delta y_1}{\Delta u_1}\right|_{u_2}$$

$$K_{11}|_{y_2} = \left.\frac{\Delta y_1}{\Delta u_1}\right|_{y_2}$$

Relative Gain Array

- for a 2x2 system there will be 4 relative gains
- for an NxN system, there will be N^2 relative gains
- these are arranged in a matrix to give the *Relative Gain Array* (RGA)

$$\Lambda = \begin{bmatrix} \lambda_{11} & \lambda_{12} & & \lambda_{1N} \\ & \ddots & & \\ & & \ddots & \\ \lambda_{N1} & \lambda_{N2} & & \lambda_{NN} \end{bmatrix} \qquad \Lambda = \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{bmatrix}$$

Elements of the RGA for 2x2 system

$$\lambda_{12} = \left. K_{12}\right|_{u_1} \Big/ \left. K_{12}\right|_{y_2} = \frac{\left.\left(\Delta y_1 / \Delta u_2 \right)\right|_{u_1}}{\left.\left(\Delta y_1 / \Delta u_2 \right)\right|_{y_2}}$$

$$\lambda_{21} = \left. K_{21}\right|_{u_2} \Big/ \left. K_{21}\right|_{y_1} = \frac{\left.\left(\Delta y_2 / \Delta u_1 \right)\right|_{u_2}}{\left.\left(\Delta y_2 / \Delta u_1 \right)\right|_{y_1}}$$

$$\lambda_{22} = \left. K_{22}\right|_{u_1} \Big/ \left. K_{22}\right|_{y_1} = \frac{\left.\left(\Delta y_2 / \Delta u_2 \right)\right|_{u_1}}{\left.\left(\Delta y_2 / \Delta u_2 \right)\right|_{y_1}}$$

Properties of RGA

- the sum of all elements in each column is unity
$$\sum_{i=1}^{N} \lambda_{ij} = 1, \; j = 1,2,\ldots,N$$
- the sum of all elements in each row is unity
$$\sum_{j=1}^{N} \lambda_{ij} = 1, \, i = 1,2,\ldots,N$$
- for an NxN system, only $(N-1)^2$ relative gains need to be determined
- relative gains are scale independent (as they are ratio of gains)

RGA for a 2x2 system

$$\Lambda = \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{bmatrix}$$

$$\lambda_{12} = 1 - \lambda_{11}$$

$$\lambda_{21} = \lambda_{12}$$

$$\lambda_{22} = \lambda_{11}$$

Analytical determination of RGA

- the RGA can also be determined analytically if a steady-state model is available
- consider the following 2x2 example:

$$y_1 = K_{11}u_1 + K_{12}u_2$$

$$y_2 = K_{21}u_1 + K_{22}u_2$$

Analytical determination of RGA

$$y_1 = K_{11}u_1 + K_{12}u_2$$

$$y_2 = K_{21}u_1 + K_{22}u_2$$

$$K_{11}\big|_{u_2} = \left.\frac{\partial y_1}{\partial u_1}\right|_{u_2} = K_{11}$$

$$y_1 = K_{11}u_1 + K_{12}\left(y_2 - K_{21}u_1\right) / K_{22}$$

$$K_{11}\big|_{y_2} = \left.\frac{\partial y_1}{\partial u_1}\right|_{y_2} = K_{11} - K_{12}K_{21} / K_{22}$$

$$\lambda_{11} = K_{11}\big|_{u_2} \Big/ K_{11}\big|_{y_2} = \frac{1}{1-\left(K_{12}K_{21}\right)/\left(K_{11}K_{22}\right)}$$

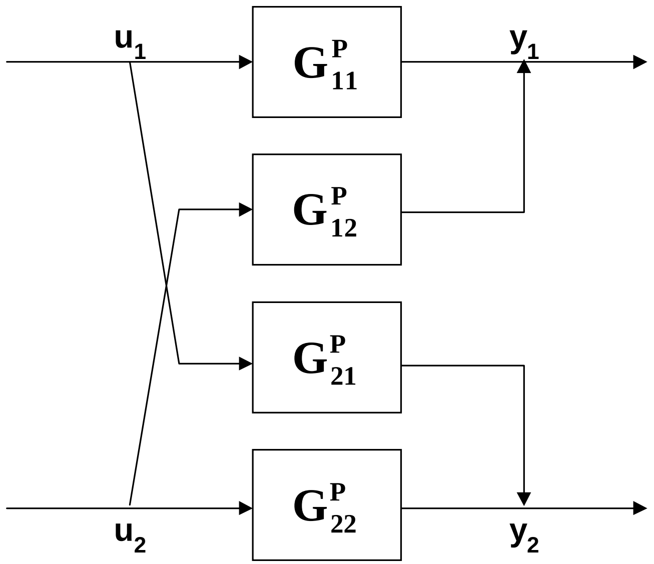
Elements of RGA for NxN system

$$\lambda_{ij} = \left. K_{ij}\right|_u \Big/ \left. K_{ij}\right|_y = \frac{\left.\left(\Delta y_i \big/ \Delta u_j\right)\right|_u}{\left.\left(\Delta y_i \big/ \Delta u_j\right)\right|_y}$$

$$\boldsymbol{\Lambda} = \mathbf{K}.*\left(\mathbf{K}^{\mathbf{T}}\right)^{-1}$$

Control loop selection based on RGA

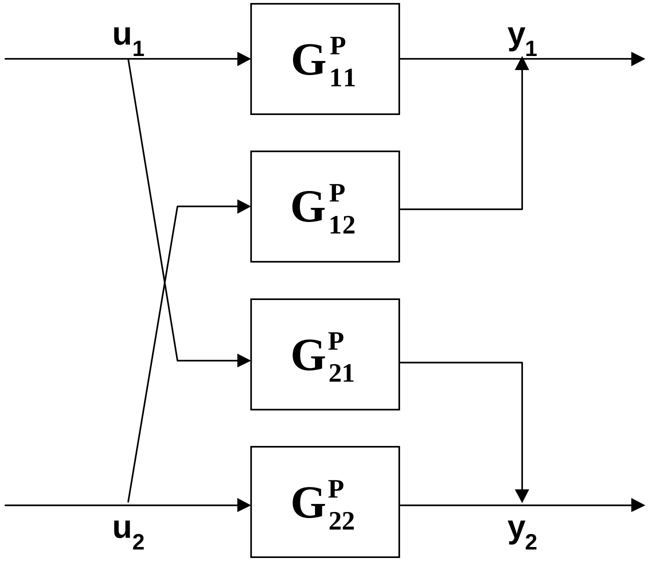
$$\lambda_{11} = 0$$



- the RGA has zero valued diagonal elements and unity off-diagonal elements.
- indicates that control of the system can only be achieved by pairing y_1-u_2, and y_2-u_1
- resulting controlled system, however, will be non-interacting.

Control loop selection based on RGA

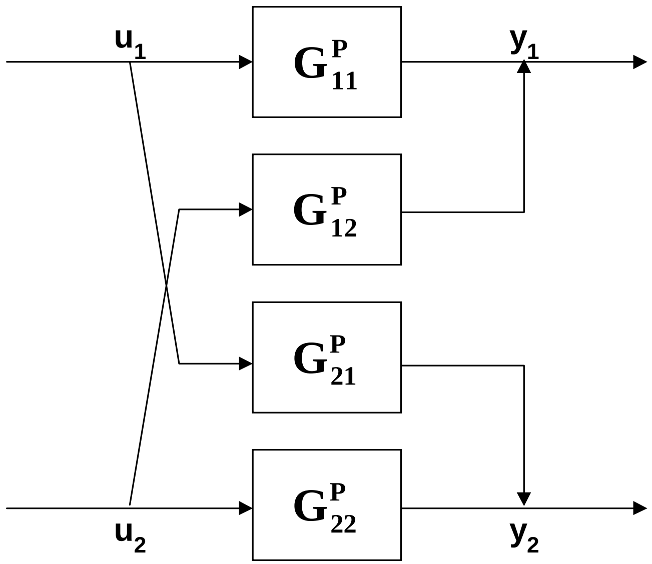
$$\lambda_{11} = 1$$



- the system is non-interacting with the pairings y_1-u_1, and y_2-u_2
- u_1 cannot be used to control y_2, nor can u_2, and be used to control y_1 (because the inputs have no effect on the respective outputs)
- best situation!

Control loop selection based on RGA

$$\lambda_{11} = 0.5$$

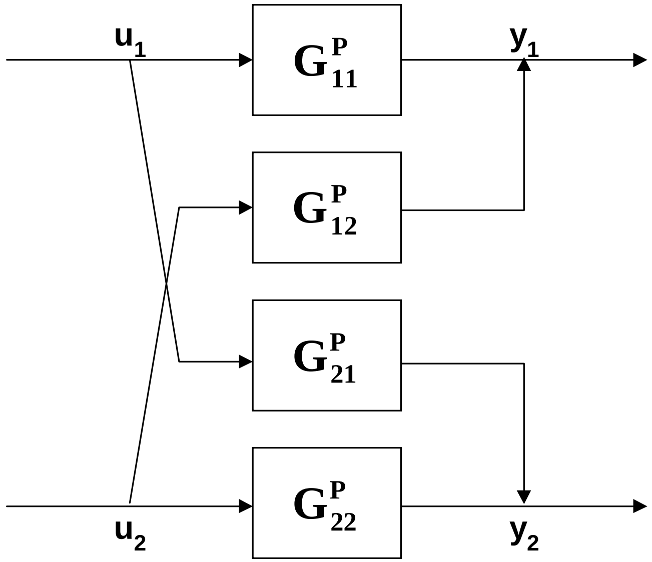


- the two manipulative inputs affect the two outputs to the same degree
- the worst case
- regardless of which pairing is used, the degree of interaction will be the same

Control loop selection based on RGA

$$0 < \lambda_{11} < 0.5$$

$$\lambda_{11} = 0.25$$

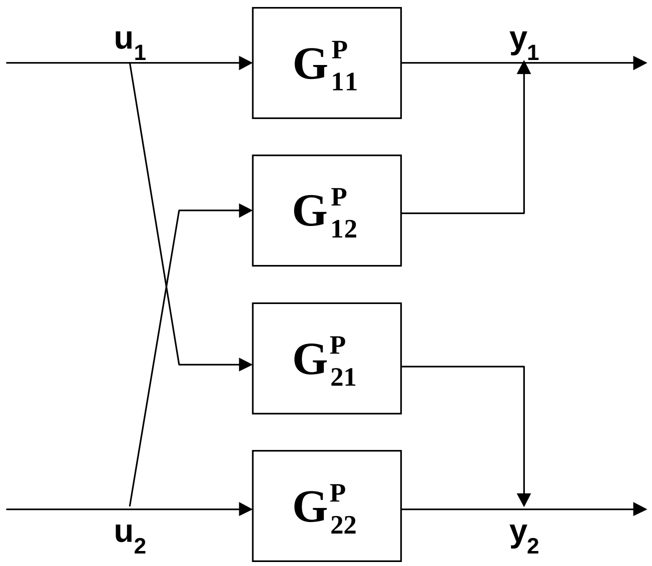


- the diagonal elements of the RGA equal 0.25 while the off-diagonal elements are 0.75
- the larger elements indicate the more suitable input-output pairings
- therefore, choose y_1-u_2, and y_2-u_1

Control loop selection based on RGA

$$0.5 < \lambda_{11} < 1$$

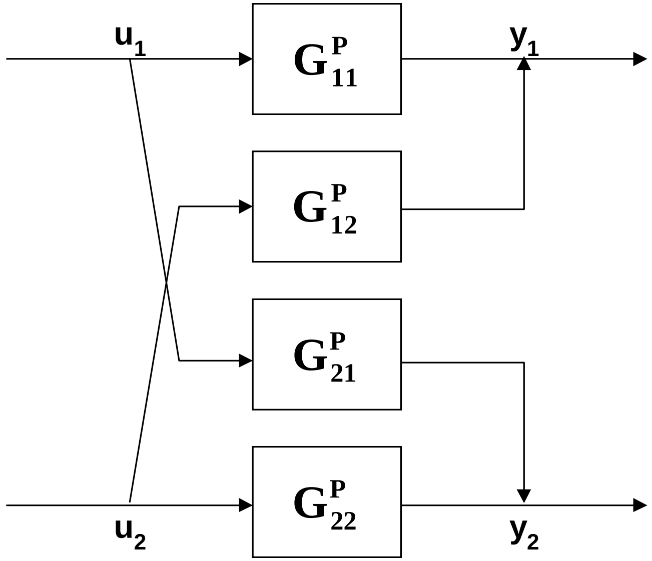
$$\lambda_{11} = 0.75$$



- this is the opposite of the previous case
- the most suitable pairings will be y_1-u_1, and y_2-u_2

Control loop selection based on RGA

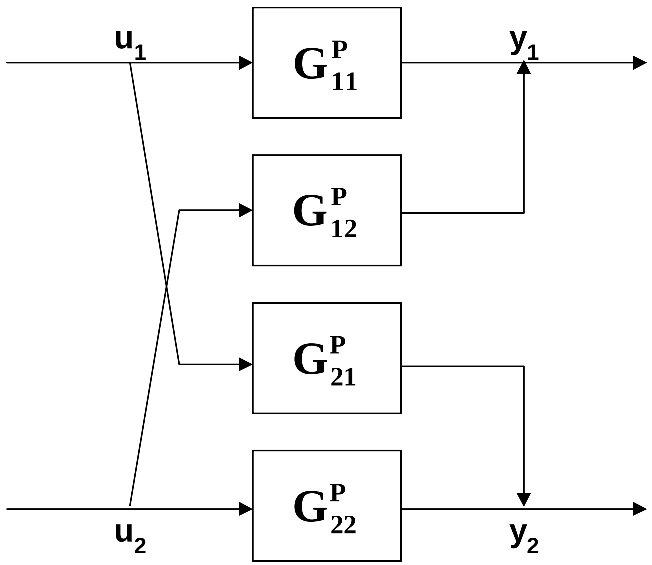
$$\lambda_{11} > 1$$



- Implies that
  - off-diagonal elements of the RGA are negative
  - note: $\lambda_{11} = K_{11}|_{u_2} / K_{11}|_{y_2} > 1$
  - if $K_{11}|_{u_2} = 1$
    then $0 < K_{11}|_{y_2} < 1$
  - change in y_1 due to a change in u_1 is reduced if the loop between y_2 and u_2 is closed

Control loop selection based on RGA

$$\lambda_{11} > 1$$



- controlled responses will be held back by the interaction
- the larger the relative gain is above unity, the larger will be this effect
- have to use larger gains
- cannot use alternate pairing as interactions will take controlled outputs in a direction away from that which the control is trying to achieve (-ve relative gain)

Limitations of RGA in loop selection

- steady-state method
- does not consider the effects of disturbances (disturbance gain array)
- not directly applicable to integrating systems
- nevertheless, still a useful technique

Control loops should have input-output pairs which give positive relative-gains that have values which are as close to unity as possible.