MULTIVARIABLE CONTROL: AN INTRODUCTION TO DECOUPLING CONTROL [†]

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INTRODUCTION

Processes with only one output being controlled by a single manipulated variable are classified as *single-input single-output* (SISO) systems. Many processes, however, do not conform to such a simple control configuration. In the process industries for example, any unit operation capable of manufacturing or refining a product cannot do so with only a single control loop. In fact, each unit operation typically requires control over at least two variables, e.g. product rate and product quality. There are, therefore, usually at least two control loops to contend with. Systems with more than one control loop are known as *multi-input multi-output* (MIMO) or *multivariable* systems. Consider the following common unit operations:

a) chemical reactors

b) distillation columns

c) heat exchangers

In reaction systems, an example of which is shown in Figure 1, the variables of interest are usually product composition and the temperature of the reaction mass.

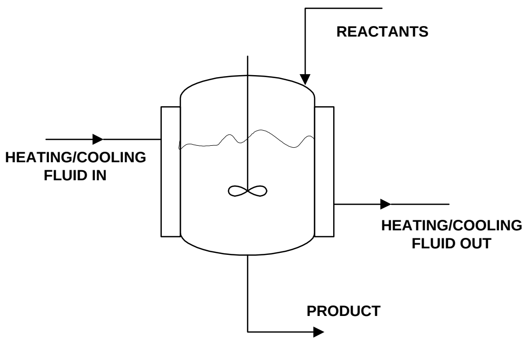

Figure 1. Schematic of a stirred tank chemical reactor

There will therefore be a composition control loop as well as a temperature control loop. Feed to the reactor is normally used to manipulate product composition while temperature is

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controlled by adding or removing energy from the reaction via heating/cooling coils or jackets. A change in feed say, to bring product composition to its desired level, would change the temperature of the reaction mass. Heat removal or addition, on the other hand, would determine the rate of reaction and hence product composition. For the two control loops to operate successfully in tandem, each loop must therefore 'know' what the other is doing. Otherwise, in trying to achieve their respective objectives, the two loops may act against the interest of the other. This phenomenon is known as *loop interaction*.

Distillation columns, which are widely used for separation and refining operations, require a phenomenal amount of energy for their operation (see Figure 2).

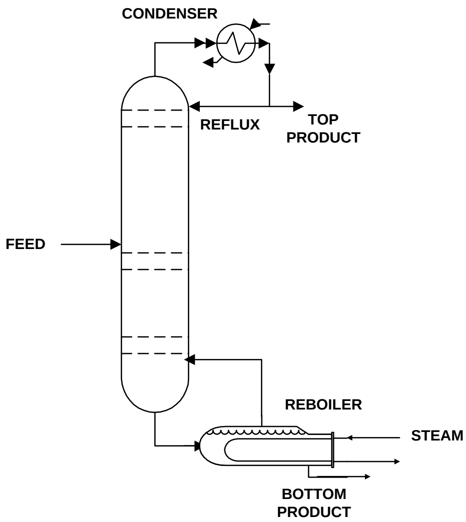

Figure 2. Schematic of distillation column

Nevertheless, minimisation of energy usage is possible if the compositions of both the top and bottom product streams are controlled to their design values, i.e. dual composition control. A common scheme is to use reflux flow to control top product composition whilst heat input is used to control bottoms product composition. However, changes in reflux also affects bottom product composition and component fractions in the top product stream are also affected by changes in heat input. Severe loop interactions can therefore occur in the dual composition control of distillation columns.

The loop interactions in the examples given above occur naturally, i.e. as a result of their physical and chemical make-up. Loop interactions may also arise as a consequence of process design: typically the use of recycle streams for heat recovery purposes. An example is where the hot bottom product stream of a distillation column is used as the heating medium to pre-heat the column feed as shown in Figure 3. Suppose heat input to the reboiler is used to control the composition of the bottom product stream. If for some reason, the composition of this stream changes, then the heat input will change in an attempt to maintain the composition at its desired level. However, changes in heat input will alter the temperature of the bottom product stream, which will then affect the temperature of the feed stream. Changes in feed temperature will in turn influence bottom product composition. It is therefore clear that interaction exists between the composition and pre-heat control loops.

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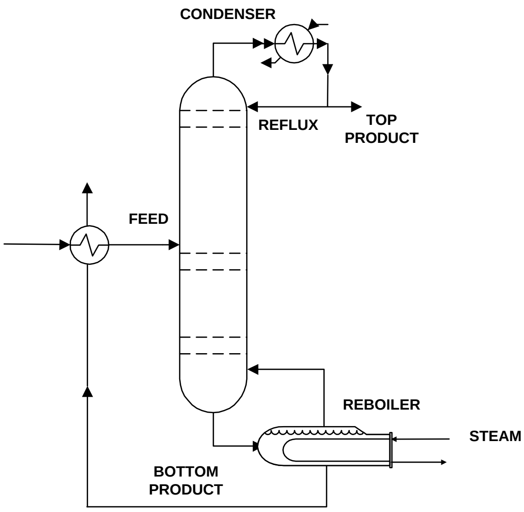


Figure 3. Distillation feed pre-heat scheme

Unless proper precautions are taken in terms of control system design, loop interactions can cause system instability. The purpose of this article is to show analytically why control loop interactions are undesirable. Two simple approaches to dealing with loop interactions are then introduced: a method for establishing the most appropriate manipulated inputs-controlled outputs pairs, and the design of multivariable control strategies that aim to eliminate interactions between control loops..

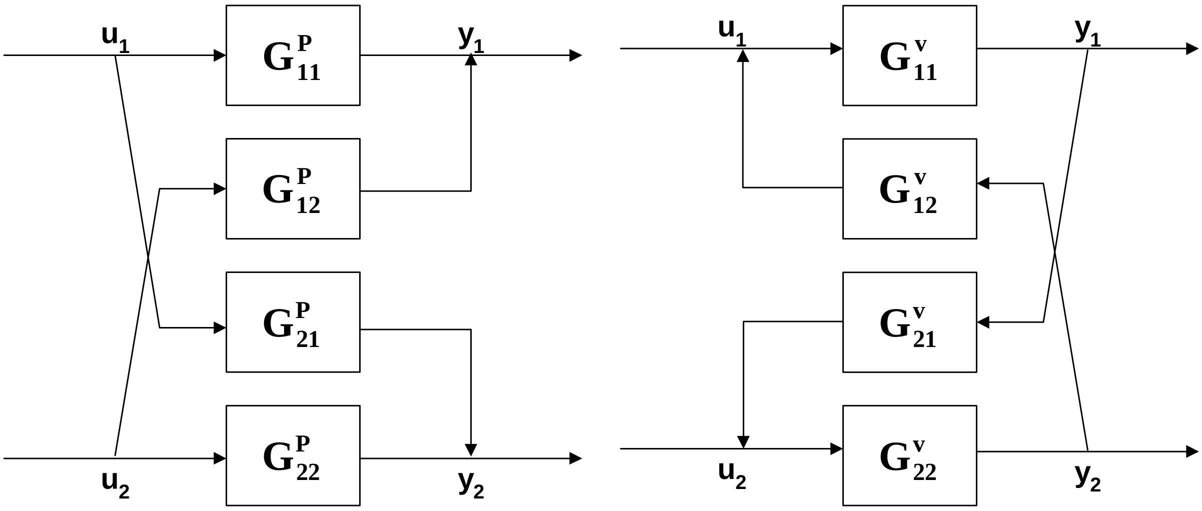
MULTIVARIABLE SYSTEMS ANALYSIS

System Representation

Established control system design techniques rely on the availability of linear system models. This is to ensure that the resulting control scheme is closely matched to the dynamics of the process. The multivariable system must therefore first be modelled, either analytically: using sets of differential equations to describe their behaviour, or empirically by fitting experimentally obtained data to an assumed structure of the process, i.e. black box modelling. Obviously, how well the resulting control strategy performs depends on the accuracy of the model. In applications where the physical and/or chemical characteristics of the system are reasonably well known, the former approach is usually adopted. Examples are in the aerospace industries where aircraft, spacecraft or missile systems can be adequately modelled by their equations of motion. In such cases, the models are usually formulated in state-space form (see e.g. Balakrishnan, 1983; O'Reilly, 1987). In the process industries, where there is a higher degree of uncertainty about process behaviour, the black-box modelling approach is often employed. However, for control systems design purposes, the input-output (transfer function) model obtained using the latter approach is generally adequate. Furthermore, there is a correspondence between state-space models and their input-output counterparts. This section therefore considers only the structures of input-output models of multivariable processes used in control systems design and analysis. More detailed treatment of multivariable system models can be found in O'Reilly (1987), Patel and Munro (1982) and in Sinha and Kuszta (1983).

Input-Output Multivariable System Models

Noting that high dimensioned systems can be decomposed to sets of (2x2) subsystems, for the sake of simplicity therefore, only a two-input two-output (2x2) process will be considered. Input-output models may assume a number of structural forms. Two common (2x2) input-output models of multivariable systems are the P- and V-canonical representations and are shown in Figure 4 below.



(a) P-canonical form (b) V-canonical form

Figure 4. Input-output Structures of 2x2 Systems

The difference between the two forms is clear from the diagrams. With the P-canonical structure, loop interactions are regarded as *feedforward* couplings whereas in the V-canonical structure, loop interactions are regarded as *feedback* couplings. The elements within the blocks of the respective diagrams are transfer functions, defining the relationship between corresponding input-output pairs.

P-canonical representation

On a loop by loop basis, the outputs are related to the inputs according to:

$$y_1 = u_1 G_{11}^p + u_2 G_{12}^p$$

$$y_2 = u_1 G_{21}^p + u_2 G_{22}^p$$

where the y_i are the systems outputs while the u_i are the manipulative inputs. The above relationships can be expressed more compactly in matrix-vector notation as:

$$\mathbf{y} = \mathbf{G}^{\mathbf{P}}\mathbf{u}$$

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where $\mathbf{y} = [y_1, y_2]^T$ and $\mathbf{u} = [u_1, u_2]^T$ while $\mathbf{G^p} = \begin{bmatrix} G_{11}^p & G_{12}^p \\ G_{21}^p & G_{22}^p \end{bmatrix}$

V-canonical representation

This MIMO representation, in contrast, has a mathematical description given by:

$$y_1 = [y_2 G_{12}^v + u_1] G_{11}^v$$

$$y_2 = [y_1 G_{21}^v + u_2] G_{22}^v$$

or in matrix-vector notation:

$$\mathbf{y} = [\mathbf{I} - \mathbf{G_m^v G_i^v}]^{-1} \mathbf{G_m^v u}$$

with $\mathbf{G_m^v} = \begin{bmatrix} G_{11}^v & 0 \\ 0 & G_{22}^v \end{bmatrix}$ and $\mathbf{G_i^v} = \begin{bmatrix} 0 & G_{12}^v \\ G_{21}^v & 0 \end{bmatrix}$

Relationship between P- and V- representations

Obviously, if a system can be modelled using both P- and V- structures, the transfer functions of both structures must be related. The transfer function matrix $\mathbf{G^p}$ of the P-canonical form is related to the V-canonical form according to:

$$\mathbf{G^p} = [\mathbf{I} - \mathbf{G_m^v G_i^v}]^{-1} \mathbf{G_m^v}$$

provided that the inverse exist.

Choice of System Representation

Given the above two process model forms, the question arises as to which representation is more useful. There are no hard and fast rules to guide the final choice. However, the following factors should be taken into account:

a) it should be possible to determine the parameters of the model from experiments

b) the model must be representative of the process, and preferably general enough to encompass other processes

c) the model should be able to provide the relevant information for control systems design

d) the model should be simple

Consider firstly, the V-canonical representation. It is not possible to obtain the elements of

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the transfer function matrices $\mathbf{G}_{\mathbf{m}}^{\mathbf{v}}$ and $\mathbf{G}_{\mathbf{i}}^{\mathbf{v}}$ from open loop step tests for example. This is because a change in one input will affect not only all outputs, the inputs are subsequently changed as well. However, the transfer functions relating to the V-canonical structure may be obtained from numerical identification techniques. Further processes are usually subject to external factors such as changes in the environment or in operating conditions. To cater for these effects, load disturbance terms may also be incorporated into the model. With the V-canonical model, the equation can be extended as follows:

$$\mathbf{y} = \left[\mathbf{I} - \mathbf{G}_{\mathbf{m}}^{\mathbf{v}}\mathbf{G}_{\mathbf{i}}^{\mathbf{v}}\right]^{-1}\left[\mathbf{G}_{\mathbf{m}}^{\mathbf{v}}\mathbf{u} + \mathbf{G}_{\mathbf{d}}\mathbf{v}\right]$$

where $\mathbf{G}_{\mathbf{d}} = \begin{bmatrix} G_{d1} & 0 \\ 0 & G_{d2} \end{bmatrix}$ and $\mathbf{v}$ is a vector of disturbance terms, $\mathbf{v} = \left[v_1, v_2\right]^T$.

Incorporation of load disturbance terms in the P-canonical representation leads to:

$$\mathbf{y} = \mathbf{G}^{\mathbf{P}}\mathbf{u} + \mathbf{G}_{\mathbf{d}}\mathbf{v}$$

Note that in this case, each input-output pair are uniquely defined. The transfer function elements of $\mathbf{G}^{\mathbf{P}}$ and $\mathbf{G}_{\mathbf{d}}$ in the P-canonical structure can therefore be directly determined from open loop experiments. An added advantage of the P-canonical representation is that the model is implicitly *observable* and *controllable*. In other words, the outputs can be measured and the inputs considered are relevant for control. Thus, this set of notes will concentrate on multivariable systems analysis and control design based on the P-canonical model.

STATEMENT OF THE INTERACTION PROBLEM

"Why do we require multivariable control?" or"What are the effects of control loop interactions on system performance?" These questions are best answered by considering the following simple example. Suppose two *independent* first order, delay free, processes G_{11} and G_{22} are under proportional control by $K_{p,1}$ and $K_{p,2}$ respectively. The characteristic equations of the two loops are then given by:

$$1 + K_{p,1}G_{11} = 0 \text{ and } \qquad 1 + K_{p,2}G_{22} = 0$$

Since a pure first order system under proportional control will always be stable, both loops will remain stable regardless of the magnitudes of the proportional gains.

Now suppose that the two loops are interacting, and let the interaction dynamics be denoted by G_{12} and G_{21} (see Fig. 4a). After some algebraic manipulation, it can be shown that the stability of the interacting system depends on the characteristic equation:

$$\left(1 + K_{p,1}G_{11}\right)\left(1 + K_{p,2}G_{22}\right) - G_{12}K_{p,2}G_{21}K_{p,1} = 0$$

This equation reveals (appearance of different signs) that the system will remain stable only

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for a range of proportional control gains. Loop interactions can therefore lead to unstable operation unless they are taken into account in the control systems design.

The problem associated with control loop interactions can now be formally stated. Following Shinskey (1979):

> *"If the steady state or dynamic gain of a given controlled variable in response to a given manipulated variable changes when other (initially open) loops are closed, then interaction exists in the system. If the controller in question was tuned with all others in manual, that tuning will be incorrect when all the others are placed in automatic because of their influence over the gain of that particular control loop. Depending on the degree of interaction, instability or at the very least, degraded responses will result."*

The problem of loop interaction can be alleviated by a proper choice of input-output pairings such that interactions will be minimised. For the (2x2) system under consideration, this is rather simple. For example, if u_1 is paired with y_2, then the other pairing is obviously u_2 with y_1. If this does not provide the required performance, then the next combination is clearly u_1 with y_1 and u_2 with y_2. Higher dimensioned systems would yield a larger combination of possible input-output pairings for control. For example, an (NxN) system will give rise to N! (N-factorial) possible input-output pairings. It is therefore important to be able to evaluate quantitatively, the degree of interaction between control loops. This information can then be used as a guide to structure a minimal interaction control scheme. The *relative gain* analysis technique is one such methodology.

THE RELATIVE GAIN ARRAY

Since its proposal by Bristol (1966), the relative gain technique has not only become a valuable tool for the selection of manipulative-controlled variable pairings, it has also been used to predict the behaviour of controlled responses (Shinskey, 1981). The analysis revolves around the construction of a *Relative Gain Array* (RGA). To fully appreciate the concept of relative gains, the RGA will be constructed for a system represented by a (2x2) P-canonical structure.

Let K_{ij} be the gains of the respective transfer functions G_{ij}. Assuming that u_2 remains constant, a step change in input u_1 of magnitude Δu_1 will produce a change Δy_1 in output y_1. Thus, the gain between u_1 and y_1 when u_2 is kept constant is given by:

$$K_{11}\big|_{u_2} = \left.\frac{\Delta y_1}{\Delta u_1}\right|_{u_2}$$

If instead of keeping u_2 constant, y_2 is now kept constant by closing the loop between y_2 and u_2. A step change in input u_1 of magnitude Δu_1 will result in another change in y_1. The gain under this new set of conditions is denoted by:

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$$K_{11}\big|_{y_2} = \left.\frac{\Delta y_1}{\Delta u_1}\right|_{y_2}$$

Although the above gains are between the same pair of variables, they may have different values because they have been obtained under different conditions. If interaction exists, then the change in y_1 due to a change in u_1 for the two cases (when u_2. and when y_2 are kept constant), will be different. The ratio,

$$\lambda_{11} = {K_{11}\big|_{u_2}} \Big/ {K_{11}\big|_{y_2}}$$

λ_{11} which is a dimensionless value, defines the *relative gain* between the output y_1 and input u_1, and immediately yields the following information:

a) If $\lambda_{11} = 0$, then a change in u_1 does not influence y_1 and therefore should not be used to control y_1.

b) If $\lambda_{11} = 1$, this implies that $K_{11}\big|_{u_2}$ and $K_{11}\big|_{y_2}$ have the same values. Thus by definition, the gain between output y_1 and input u_1 is not affected by the loop between y_2 and u_2, i.e. interaction does not exist.

For a (2x2) process, there are three other relative gain elements, namely :

$$\lambda_{12} = {K_{12}\big|_{u_1}} \Big/ {K_{12}\big|_{y_2}} = \frac{\left.\left(\Delta y_1 / \Delta u_2\right)\right|_{u_1}}{\left.\left(\Delta y_1 / \Delta u_2\right)\right|_{y_2}}$$

$$\lambda_{21} = {K_{21}\big|_{u_2}} \Big/ {K_{21}\big|_{y_1}} = \frac{\left.\left(\Delta y_2 / \Delta u_1\right)\right|_{u_2}}{\left.\left(\Delta y_2 / \Delta u_1\right)\right|_{y_1}}$$

$$\lambda_{22} = {K_{22}\big|_{u_1}} \Big/ {K_{22}\big|_{y_1}} = \frac{\left.\left(\Delta y_2 / \Delta u_2\right)\right|_{u_1}}{\left.\left(\Delta y_2 / \Delta u_2\right)\right|_{y_1}}$$

Finally, the matrix of elements λ_{ij} make up the *Relative Gain Array* (RGA):

$$\Lambda = \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{bmatrix}$$

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For a general system with N-inputs and N-outputs, there will be (NxN) relative gain elements and the relative gains between an output y_i and an input u_j are given by:

$$\lambda_{ij} = \frac{K_{ij}|_u}{K_{ij}|_y} = \frac{\left.\left(\frac{\Delta y_i}{\Delta u_j}\right)\right|_u}{\left.\left(\frac{\Delta y_i}{\Delta u_j}\right)\right|_y}$$

The subscripts u and y denotes constant values of $u_m, m \neq j$; $y_n, n \neq i$ respectively, and the RGA for the (NxN) system is:

$$\Lambda = \begin{bmatrix} \lambda_{11} & \lambda_{12} & & \lambda_{1N} \\ & \ddots & & \\ & & \ddots & \\ \lambda_{N1} & \lambda_{N2} & & \lambda_{NN} \end{bmatrix}$$

It may appear that the determination of all the RGA elements is a tedious and cumbersome task. In practice however, this is not so since the elements of the RGA have the following properties:

a) The sum of all elements in each column is unity, i.e. $\sum_{i=1}^{N} \lambda_{ij} = 1, \; j = 1,2,\ldots,N$

b) The sum of all elements in each row is unity, i.e. $\sum_{j=1}^{N} \lambda_{ij} = 1, \; i = 1,2,\ldots,N$

Thus, for a (2x2) system, determination of one RGA element uniquely defines the other three, i.e.

$\lambda_{12} = 1 - \lambda_{11}$ $\quad \lambda_{21} = \lambda_{12}$ $\quad$ and $\quad \lambda_{22} = \lambda_{11}$

Similarly, for an (NxN) system, only (N-1)*(N-1) elements need to be determined.

The above method of constructing the RGA relates to an experimental procedure. An analytical determination is possible if a steady-state model of the system is available. Thus if:

$y_1 = K_{11}u_1 + K_{12}u_2$ and

$y_2 = K_{21}u_1 + K_{22}u_2$

where the K_{ij} are the steady state gains of the process transfer function matrix. Then

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$$K_{11}|_{u_2} = \left.\frac{\partial y_1}{\partial u_1}\right|_{u_2} = K_{11}$$

Eliminating u_2 from the steady-state relationships yields:

$$y_1 = K_{11}u_1 + K_{21}\left(y_2 - K_{21}u_1\right) / K_{22}$$

Differentiating this w.r.t. u_1 while keeping y_2 constant then yields:

$$K_{11}|_{y_2} = \left.\frac{\partial y_1}{\partial u_1}\right|_{y_2} = K_{11} - K_{12}K_{21} / K_{22}$$

The relative gain λ_{11} is therefore given by:

$$\lambda_{11} = {K_{11}|_{u_2}}\big/{K_{11}|_{y_2}} = \frac{1}{1-\left(K_{12}K_{21}\right)/\left(K_{11}K_{22}\right)}$$

The remaining RGA elements of the (2x2) system then follows.

For a general (NxN)) system, provided that the process gain matrix is known, e.g. $\mathbf{K}$, then the RGA can be calculated as:

$$\Lambda = \mathbf{K}.*\left(\mathbf{K}^{\mathbf{T}}\right)^{-1}$$

where the '.*' operator implies an element by element multiplication.

Selecting Control Loops using the RGA

Suppose the RGA has been constructed. The following cases may arise:

a) If $\lambda_{11} = 0$, then the RGA has zero valued diagonal elements and unity off-diagonal elements. This indicates that control of the system can only be achieved by pairing y_1 with u_2, and y_2 with u_1. The resulting controlled system, however, will be non-interacting.

b) If $\lambda_{11} = 1$, then the system is non-interacting with the pairings y_1-u_1 and y_2-u_2. By the same token, u_1 cannot be used to control y_2, nor can u_2 be used to control y_1 because the inputs have no effect on the respective outputs.

c) If $\lambda_{11} = 0.5$, then the two manipulative inputs affect the two outputs to the same degree. This reflects the worst case; regardless of which pairing is used, the degree of interaction will be the same.

d) If $0 < \lambda_{11} < 0.5$ ($\lambda_{11} = 0.25$ say), then the diagonal elements of the RGA equal

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0.25 while the off-diagonal elements are 0.75. The larger elements indicate the more suitable input-output pairings, viz. y_1 with u_2, and y_2 with u_1.

e) If $0.5 < \lambda_{11} < 1$ ($\lambda_{11} = 0.75$ say), then the opposite of case (d) occurs and the most suitable pairings will be y_1-u_1 and y_2-u_2.

f) If $\lambda_{11} > 1$, then the off-diagonal elements of the RGA will be negative. If the pairings y_1-u_1 and y_2-u_2.are employed, the corresponding relative gains are therefore larger than unity. By definition, this implies:

$$\lambda_{11} = K_{11}|_{u_2} / K_{11}|_{y_2} > 1 \qquad \text{and} \qquad \lambda_{22} = K_{22}|_{u_1} / K_{22}|_{y_1} > 1$$

Suppose $K_{11}|_{u_2} = 1$, then for $\lambda_{11} > 1$, $0 < K_{11}|_{y_2} < 1$. What this means is that the change in y_1 due to a change in u_1 is reduced if the loop between y_2 and u_2 is closed. In other words, controlled responses will be held back by the interaction from the other loop. The larger the relative gain is above unity, the larger will be this effect. The mentioned pairings will therefore require the use of large controller gains. The alternate pairings y_1 with u_2, and y_2 with u_1.are, however, unsuitable because the corresponding relative gains are negative. This means that the resulting interactions will take controlled outputs in a direction away from that which the control is trying to achieve. As a result, control will eventually be lost.

The above cases, together with the corresponding rules for input-output selection, is far from complete. A rigorous analysis would include consideration of systems where there are more inputs than outputs, e.g. the presence of disturbances. The signs of the process gains are also important factors in analysing the RGA. These are outside the scope of this article. Interested readers are referred to the publications by Bristol (1966, 1978) and Shinskey (1977, 1979). Nevertheless, despite the incompleteness of the cases considered, a useful and general rule for the selection of control loops can be extracted:

> *Control loops should have input-output pairs which give positive relative-gains that have values which are as close to unity as possible.*

The use of relative gains to determine the best manipulative input-controlled output pairings for multivariable control leads to what is termed *dominant interaction* control strategy. There are other techniques, such as those based on characteristic loci decomposition and frequency domain based methodologies, which also attempt to achieve minimum interaction between control loops [see e.g. Patel and Munro (1982); O'Reilly (1987) and the reviews by Rijnsdorp and Seborg (1979) and by Schwanke *et al* (1977)]. Although techniques based on analyses of relative gains are essentially steady-state design procedures, in process control situations however, they have proved to be quite popular because of the intuitive nature of these approaches.

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MULTIVARIABLE DECOUPLING CONTROL

Concepts

Another popular approach to dealing with control loop interactions is to design *non-interacting* or *decoupling* control schemes. Here, the objective is to eliminate completely the effects of loop interactions. This is achieved via the specification of compensation networks known as *decouplers*. Essentially, the role of decouplers is to decompose a multivariable process into a series of independent single-loop sub-systems. If such a situation can be achieved, then complete or ideal decoupling occurs and the multivariable process can be controlled using independent loop controllers. The diagram below shows the general decoupling control structure.

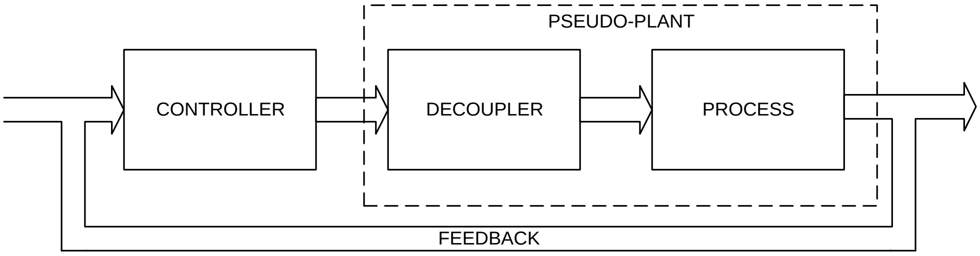

Figure 5. General structure of Decoupling Control System

The form of the decoupling network has been the cause of some controversy in the past. As with the input-output representation of multivariable processes, different structures are possible, e.g. P- or V-decouplers. Judging by the literature however, the P-decoupler seems to be the most popular. This corresponds to the structure shown in Figure 4a. From this, two decoupling networks may be derived.

The decoupling network of Boksenbom and Hood (1949):

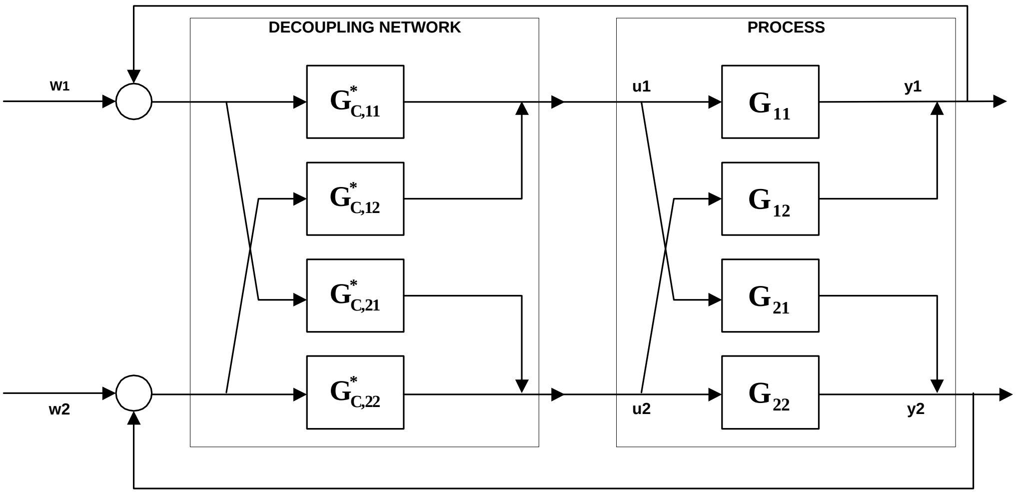

Figure 6. Decoupling control system (Boksenbom and Hood, 1949)

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The decoupling network structure of Boksenbom and Hood (1949) is shown in Figure 6. Let $\mathbf{G}_{\mathbf{c}}^{*}$ be the matrix of decoupling elements:

$$\mathbf{G}_{\mathbf{c}}^{*} = \begin{bmatrix} G_{c,11}^{*} & G_{c,12}^{*} \\ G_{c,21}^{*} & G_{c,22}^{*} \end{bmatrix}$$

and let the (2x2) process transfer function matrix $\mathbf{G}$ be as described before, along with the associated input and output vectors $\mathbf{u}$ and $\mathbf{y}$ respectively. Introducing a vector of set-points or reference signals, $\mathbf{w} = [w_1, w_2]^{\mathbf{T}}$, results in the following equations:

$$\mathbf{Gu} = \mathbf{y} \qquad \mathbf{u} = \mathbf{G}_{\mathbf{c}}^{*}[\mathbf{w} - \mathbf{y}] \quad \mathbf{GG}_{\mathbf{c}}^{*}[\mathbf{w} - \mathbf{y}] = \mathbf{y}$$

Rearrangement yields the closed loop expression:

$$\mathbf{y} = \left[\mathbf{I} + \mathbf{GG}_{\mathbf{c}}^{*}\right]^{-1} \mathbf{GG}_{\mathbf{c}}^{*}\mathbf{w}$$

For the individual loops of the closed loop system are independent of one another, it is required that:

$$\mathbf{X} = \left[\mathbf{I} + \mathbf{GG}_{\mathbf{c}}^{*}\right]^{-1} \mathbf{GG}_{\mathbf{c}}^{*} = diag[x_1, x_2]$$

ie. **X** must be a diagonal matrix. Since the sum and product of two diagonal matrices are diagonal matrices, and the inverse of a diagonal matrix is also a diagonal matrix, then the requirement can be ensured if $\mathbf{GG}_{\mathbf{c}}^{*}$ is made diagonal, i.e.

$$\mathbf{GG}_{\mathbf{c}}^{*} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} G_{c,11}^{*} & G_{c,12}^{*} \\ G_{c,21}^{*} & G_{c,22}^{*} \end{bmatrix} = \begin{bmatrix} q_1 & 0 \\ 0 & q_2 \end{bmatrix}$$

$$= \begin{bmatrix} G_{11}G_{c,11}^{*} + G_{12}G_{c,21}^{*} & G_{11}G_{c,12}^{*} + G_{12}G_{c,22}^{*} \\ G_{21}G_{c,11}^{*} + G_{22}G_{c,21}^{*} & G_{21}G_{c,12}^{*} + G_{22}G_{c,22}^{*} \end{bmatrix} = \begin{bmatrix} q_1 & 0 \\ 0 & q_2 \end{bmatrix}$$

Comparing each element of the matrices results in a set of four equations:

$$q_1 = G_{11}G_{c,11}^{*} + G_{12}G_{c,21}^{*}$$

$$0 = G_{11}G_{c,12}^{*} + G_{12}G_{c,22}^{*}$$

$$0 = G_{21}G_{c,11}^{*} + G_{22}G_{c,21}^{*}$$

$$q_2 = G_{21}G_{c,12}^{*} + G_{22}G_{c,22}^{*}$$

Thus if the transfer function elements of $\mathbf{G}$ are known, and having specified the diagonal

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elements of $\mathbf{G}^*_\mathbf{c}$, then the appropriate off-diagonal elements of $\mathbf{G}^*_\mathbf{c}$ to achieve decoupling control can be calculated by solving the above set of equations. The simplest way to do this is to concentrate on the conditions that will set $\mathbf{G}\mathbf{G}^*_\mathbf{c}$ to a diagonal matrix, i.e. the above equations that are equal to zero. These give the following relationships:

$$G^*_{c,12} = \frac{-G_{12}G^*_{c,22}}{G_{11}} \quad \text{and} \quad G^*_{c,21} = \frac{-G_{21}G^*_{c,11}}{G_{22}}$$

If the forward path decoupling elements, $G^*_{c,11}$ and $G^*_{c,22}$, are taken to be PI controllers say, then given the transfer functions of the process, the decoupling network will be fully specified. This design technique requires a knowledge of the process transfer function matrix **G**, and therefore emphasises the usefulness of representing the process in P-canonical form. As mentioned previously, the transfer functions of the P-canonical process representation can be obtained experimentally.

Note, however, that this decoupling control structure addresses the servo problem only. The rejection of load disturbances requires a different approach. With this configuration, if the forward path compensation elements, viz. $G^*_{c,11}$ and $G^*_{c,22}$, are tuned on-line, then the off-diagonal compensation elements must be recalculated. Whilst this may not be a major problem if the strategy is being implemented on a microprocessor, the inter-dependence of the decoupling elements becomes a distinct disadvantage when one of the loops is placed under manual control. The decoupling effect will then be lost.

The method of Zalkind (1967) and Luyben (1970):

The previous discussion highlights the need for a decoupling network which should be independent of loop controllers. The following diagram shows one such decoupling control scheme.

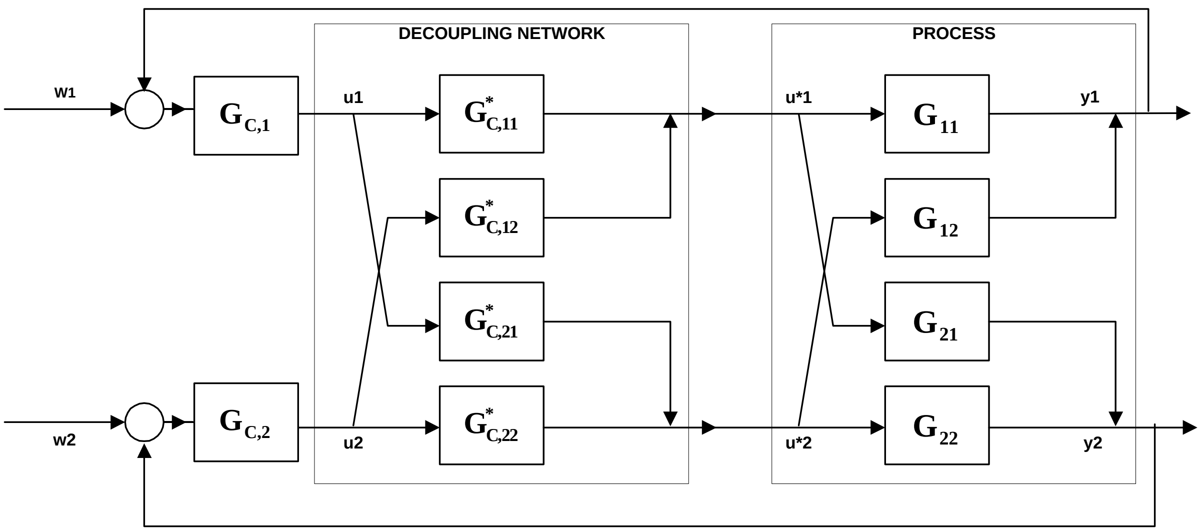

Figure 7. Non-interacting decoupling control structure (Zalkind, 1967; Luyben, 1970)

Here, in addition to the decoupling network, there are two extra blocks which represent the forward path controllers. In contrast to the previous strategy, the decoupling network forms the secondary post-compensation block and allows more flexibility in the implementation and commissioning of the non-interacting control scheme. Let the forward path control matrix be denoted $\mathbf{G_c}$ with output $\mathbf{u}$, and the output of the decoupling network be denoted $\mathbf{u}^*$. The system is described by the following relationships:

$$\mathbf{y} = \mathbf{G}\mathbf{u}^* \qquad \mathbf{u}^* = \mathbf{G}_c^*\mathbf{u} \qquad \mathbf{u} = \mathbf{G_c}[\mathbf{w} - \mathbf{y}]$$

Therefore

$$\mathbf{y} = \mathbf{G}\mathbf{G}_c^*\mathbf{u} = \mathbf{G}\mathbf{G}_c^*\mathbf{G_c}[\mathbf{w} - \mathbf{y}]$$

Recall that the objective is to artificially create a situation where the forward path controllers ‘think’ that they are controlling two independent loops. Since $\mathbf{G_c}$ is a diagonal matrix, the objective will be achieved if:

$$\mathbf{X} = \mathbf{G}\mathbf{G}_c^* = diag[x_1, x_2]$$

To determine $\mathbf{G}_c^*$, we have to calculate the inverse of $\mathbf{G}$ since:

$$\mathbf{G}_c^* = \mathbf{G}^{-1}\mathbf{X} \quad \text{with} \quad \mathbf{G}^{-1} = \text{adj}(\mathbf{G})/\det(\mathbf{G})$$

where $\text{adj}(\mathbf{G})$ and $\det(\mathbf{G})$ denote the *adjoint* and the *determinant* of the matrix $\mathbf{G}$ respectively, and they are given by:

$$\det(\mathbf{G}) = G_{11}G_{22} - G_{12}G_{21}$$

$$\text{adj}(\mathbf{G}) = \begin{bmatrix} G_{22} & -G_{12} \\ -G_{21} & G_{11} \end{bmatrix}$$

Since $\mathbf{X} = diag[x_1, x_2]$, therefore

$$\mathbf{G}_c^* = \mathbf{G}^{-1}\mathbf{X} = \begin{bmatrix} G_{22}x_1 & -G_{12}x_2 \\ -G_{21}x_1 & G_{11}x_2 \end{bmatrix} \frac{1}{\det(\mathbf{G})}$$

The simplest form of this decoupling matrix has unity diagonal elements, i.e.:

$$G_{c,11}^* = G_{c,22}^* = 1$$

which leads to the following off-diagonal elements:

$$G_{c,12}^* = -G_{12} / G_{11} \qquad \text{and} \qquad G_{c,21}^* = -G_{21} / G_{22}$$

The above equations show that with this method, the decoupling elements are independent of the forward path controllers. On-line tuning of the controllers, therefore, does not require redesign of the decoupling elements; controller modes may be changed say from PI to PID and either of the forward path controllers may be placed in manual without loss of decoupling. Note also, that decoupling occurs between the forward path control signals and the process outputs, and not between set-points and process outputs. This technique is therefore not restricted to the servo problem. As with the previous method, however, the process transfer function matrix still has to be known.

Implementation Considerations

The more astute would have noticed that decoupling elements bear strong similarities to dynamic 'trim' mechanisms or feedforward compensators. Indeed, the difficulties of realising feedforward compensation apply in decoupling control (see e.g. Wardle and Wood, 1969). Theoretical decoupling is based on an assumption of linear processes and that exact cancellation of numerator and denominator dynamics of interaction terms can be achieved. This immediately precludes application to systems with non-minimum phase behaviour, i.e. systems with unstable zeros. Since the cancellation procedure translates zeros to poles, unstable decoupling elements may result. A similar problem occurs if erroneous models are used. Cancellation will then be incomplete and may result in unstable closed loop poles. Even if an accurate model of the process is available, high order dynamics may render the realisation too complex.

A more specific problem of realisability relates to the time-delays associated with the elements of the process transfer function matrix (e.g. Niederlinski, 1971; Waller 1974). Notice that the above decoupling techniques involve the ratios G_{12}/G_{11} and G_{21}/G_{22}. Suppose that:

$$G_{12} = \frac{K_{12}\exp(-\theta_{12})}{1+\tau_{12}s} \quad \text{and} \quad G_{11} = \frac{K_{11}\exp(-\theta_{11})}{1+\tau_{11}s}$$

where θ_{ij} and τ_{ij} denote time-delays and time-constants respectively, then:

$$\frac{G_{12}}{G_{11}} = \frac{K_{12}\left(1+\tau_{11}s\right)}{K_{11}\left(1+\tau_{12}s\right)}\exp(\theta_{11}-\theta_{12})$$

If $\theta_{11} > \theta_{12}$, the argument of the exponential term will be positive, implying that future values of process variables are needed for implementation.

Due to the above mentioned difficulties, ideal decouplers are seldom employed. In many cases, the engineering approach of simplification was adopted, achieving results which were surprisingly better than those obtained using the more rigorous approach of ideal decoupling. Referring to the time-delay example, a first approximation may be to neglect time-delay effects in calculating decoupling elements. The problem of high order dynamics may also be alleviated by designing decoupling networks based on a reduced order model of the process. Partial decoupling may be employed if the effect of one of the interaction terms is deemed

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negligible. This reduces the decoupling problem to one very similar to feedforward compensation. A more drastic simplification is to ignore the dynamics completely and to rely on static-decouplers. Here, the process transfer functions G_{ij}'s are approximated by the gains K_{ij}'s. In doing so, the problem of non-minimum phase behaviour is avoided. For further details, the reader is referred to the articles by Shinskey (1977), McAvoy (1979), Luyben (1970) and Fagervik *et al* (1981).

Combining Non-interacting and Dominant-interaction Designs

Very often, the non-interacting design is preceded by a relative gain analysis to determine the most suitable input-output pairings. If interactions still exist, then a suitable decoupling network is designed. The combination of the two approaches to multivariable control yields a more robust strategy. The reason for this is intuitive. Input-output pairings chosen to reduce interactions not only mean that the resulting decoupling compensators need to 'do less work', it also means that there is a larger allowable margin of error in decoupler design. The second benefit, which is usually discounted, is that the determination of relative gains implicitly forces the control engineer to perform a process analysis. This will lead to a better understanding of process behaviour, and will naturally result in the formulation of a better control strategy. A possible design procedure can now be summarised as follows:

a) determine relative gains of process.

b) choose manipulative input-controlled output pairs such that interactions will be minimised.

c) if necessary, design the appropriate decoupling network to deal with residual interactions.

SUMMARY

The field of multivariable control is very wide and an introductory article of this nature cannot possibly cover all aspects of the topic. This is merely an attempt to draw attention to the consequences of interacting control loops and the utility of two relatively simple techniques for alleviating these problems, viz. dominant interaction control systems design using relative gain analyses and the use of decoupling networks to provide for non-interacting multivariable control. Finally, although the examples used have been drawn from the process industries, it should be recognised that the techniques described are applicable to multivariable systems in general.

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