DYNAMIC MODELS FOR CONTROLLER DESIGN

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1. INTRODUCTION

The problem of designing a good control system is basically that of matching the dynamic characteristics of a process by those of the controller. In other words, if the dynamics of the process and the characteristics of the disturbances affecting it are known, then the controller that will provide the desired closed loop performance can be designed. 'Modern' approaches to controller design therefore presuppose that a suitable description (model) of the process to be controlled is available. The model could be in the form of statistical relationships; continuous time differential equations; difference equations; continuous or discrete transfer functions; time-series; even qualitative descriptions such as qualitative transfer functions and rule based models. However, due to the difficulties involved in developing physical-chemical models, relatively simple, lumped parameter, input-output (black-box) models are often used. With few exceptions, current process control systems are also implemented on sampled data devices. This set of notes will therefore focus only on the relevant model forms that can be used for the synthesis of discrete model based controllers, namely transfer function and time-series models.

2. DISCRETE-TIME TRANSFER FUNCTION MODELS

2.1 Single-input Single-output (SISO) Linear Systems

Consider a continuous time process with the Laplace transfer function:

$$\frac{Y(s)}{U(s)} = G(s) = \frac{\overline{B}(s)e^{-T_d s}}{\overline{A}(s)} \tag{1}$$

relating the effects of input $U(s)$ on output $Y(s)$. $\overline{A}(s)$ and $\overline{B}(s)$ are polynomials in the Laplace operator 's', describing respectively, the numerator and denominator dynamics of the process. T_d is the time delay. Upon introduction of a zero-order-hold (ZOH) and sampling, the equivalent discrete transfer function[1] is:

$$Z\{ZOH(s).G(s)\} = \frac{Y(z)}{U(z)} = HG(z) = \frac{B(z)z^{-k}}{A(z)} \qquad (2)$$

where $k \geq 1$ is the time delay of the discretised process, expressed as an integer multiple of the sampling interval T_s and $A(z)$ and $B(z)$ are polynomials in z^{-1}. That is:

$$k = \mathrm{int}\left(\frac{T_d}{T_s}\right) + 1 \qquad (3a)$$

$$A(z) = 1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_{N_A} z^{-N_A}, \quad N_A = \deg(A(z)) \qquad (3b)$$

$$B(z) = b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots + a_{N_B} z^{-N_B}, \quad N_B = \deg(B(z)) \qquad (3c)$$

It is easy to convert Eq.(2) into a difference equation, for the purposes of simulation say. Simply introduce $y(t)$ and $u(t)$ in place of $Y(z)$ and $U(z)$ respectively, and regard z^{-1} as a time shift operator[2]. Hence, the difference equation form of Eq.(2) is written as,

$$A(z)y(t) = B(z)u(t-k)$$

or

$$\left(1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_{N_A} z^{-N_A}\right) y(t) = \left(b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_{N_B} z^{-N_B}\right) u(t-k)$$

or

$$\begin{aligned} y(t) = &-a_1 y(t-1) - a_2 y(t-2) - \cdots - a_{N_A} y(t-N_A) + \\ &b_0 u(t-k) + b_1 u(t-k-1) + b_2 u(t-k-2) + \cdots + b_{N_B} u(t-k-N_B) \end{aligned} \qquad (4)$$

Here, 't' denotes the time index, and the operation of z^{-n} on a time variable $x(t)$ results in

$$z^{-n} x(t) = x(t-n) \qquad (5)$$

that is, $x(t)$ is shifted n sample intervals back in time. It is important to remember that we are working with transfer functions, where all initial conditions are assumed to be zero. In other words, the time indexed variable $y(t)$ and $u(t)$ are actually deviation variables, i.e.

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[1] Appendix 1 and Appendix 2 contain introductory material on sampled-data-systems and z-transforms

[2] In some publications, q^{-1} is used as the time-shift operator instead of z^{-1} to avoid confusion between the complex z operator in the discrete z-domain.

$$y(t) = Y(t) - Y(\infty)$$

and $$u(t) = U(t) - U(\infty)$$

where $Y(t)$ and $U(t)$ are the measured output and input signals while $Y(\infty)$ and $U(\infty)$ are some corresponding compatible steady-states: a steady input $U(\infty)$ will cause the output to settle at $Y(\infty)$. This has significant implications not only from the modelling point of view but also from the parameter estimation perspective as will be discussed in detail later. It is also important to bear in mind that theoretically, transfer-functions are capable of representing linear-systems only.

Instead of expressing the discrete transfer function as a ratio of two polynomials, we can also write

$$\frac{Y(z)}{U(z)} = \frac{B(z)z^{-k}}{A(z)} = z^{-k}\beta(z) \qquad (6)$$

where $\beta(z)$ is another polynomial in z^{-1}, the result of dividing $B(z)$ by $A(z)$. In this case, the difference equation relating $y(t)$ to $u(t)$ becomes

$$y(t) = \beta_0 u(t-k) + \beta_1 u(t-k-1) + \beta_2 u(t-k-2) + \cdots \qquad (7)$$

Thus, the output is dependent only on a weighted sum of previous inputs. Depending on the nature of $B(z)$ and $A(z)$, the right-hand-side of Eq.(7) may be an infinite series in u. This representation is useful because if $u(t\text{-}k)$ is a unit impulse, then the coefficients of $\beta(z)$ defines the magnitudes of the resultant output sequence.

Consider the following example

$$y(t) = 0.5u(t-3) + 0.7u(t-4) + 0.4u(t-5) + 0.3u(t-6) + 0.1u(t-7) \qquad (8)$$

which has a delay of 3 sample intervals. Figure 1 shows what happens when it is subject to an unit impulse input (dashed line) at sample number 12.

After the passage of the time delay, i.e. at sample number 15, the system output is 0.5. At sample 16, the output is 0.7, and so on. It can be seen that the output trajectory matches exactly the weights on the input sequence of Eq.(8). What this means is that if the process was subject to a unit impulse, then the coefficients of $\beta(z)$ could be determined from the output measurements.

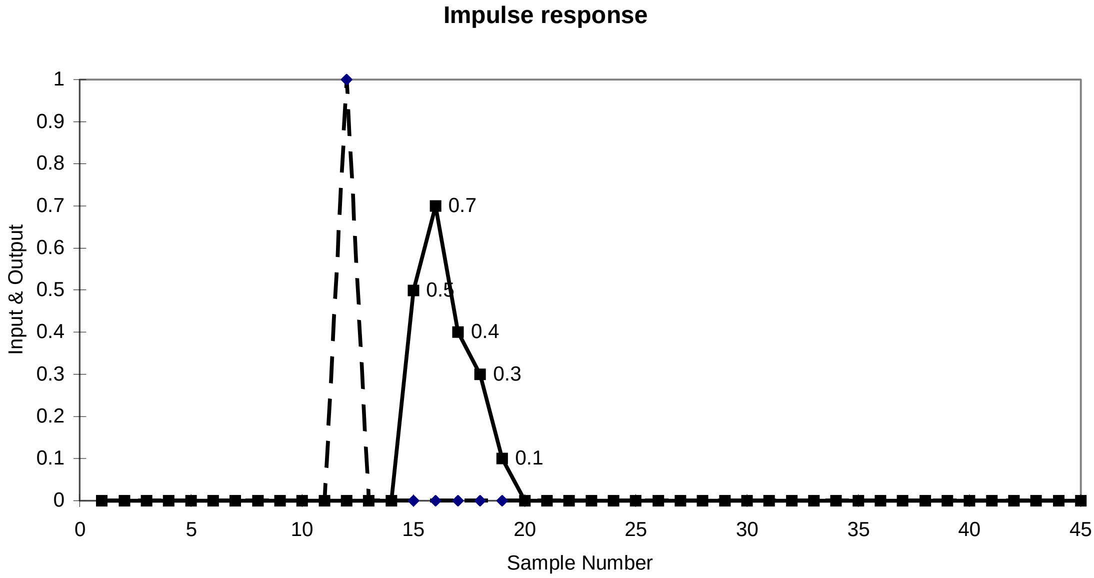

Figure 1. Impulse response of system given by Eq.(8)

The response of the system when it is subject to a unit step input (dashed line) is shown in Fig. 2.

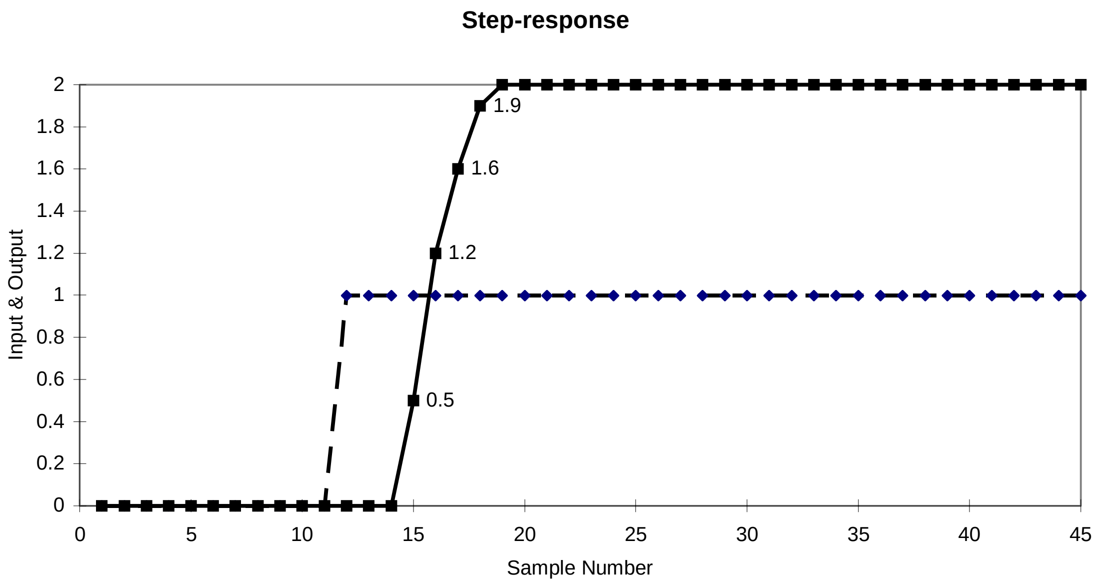

Figure 2. Step response of system given by Eq.(8)

In this case, we see that the output response trajectory is given by successive convolution of the coefficients of Eq.(8). That is, at sample number 15, the output is at 0.5. It then moves to ($\beta_0 + \beta_1 = 0.5+0.7 = 1.2$) followed by ($\beta_0 + \beta_1 + \beta_2 = 0.5+0.7+0.4 = 1.6$) and so on.

Although models with structures given by Eq.(2) are more compact, Eq.(6) is more versatile. Since they are easily parameterised by simple experiments, they can be used to model systems with 'irregular' response behaviour.

2.2 Multi-input Single-output Representations

Both Eqs.(2) and (6) can be extended to include the effects of other deterministic inputs on the process output. For example, if the process is affected by a load disturbance, $v(t)$, then Eq.(2) can be modified to

$$Y(z) = HG(z).U(z) + G_D(z).V(z) = \frac{B_1(z)z^{-k}}{A_1(z)}U(z) + \frac{B_2(z)z^{-l}}{A_2(z)}V(z) \qquad (9a)$$

which can then be expressed as

$$A(z)Y(z) = z^{-k}B(z)U(z) + z^{-l}D(z)V(z) \qquad (9b)$$

'l' is the delay between the disturbance and the output, again expressed as an integer multiple of the sample interval and $D(z)$ is another polynomial in z^{-1} However, unlike 'k' the delay between the manipulated input and output which has a minimum value of 1 due to the presence of the ZOH, $l \geq 0$. This is because there is no hold device associated with the disturbance input. If there are more inputs, they can be included in a similar manner. Likewise, Eq.(6) can be modified to include inputs other than the manipulated variable, e.g.

$$Y(z) = z^{-k}\beta(z)U(z) + z^{-l}\gamma(z).V(z) \qquad (10)$$

where $\gamma(z)$ is a polynomial in z^{-1}.

As with continuous time systems, a set of transfer function relationships can be combined in vector-matrix form to described multivariable systems, i.e. systems with more than one output and output.

3. Time-series Models

Next we consider the time-series approach to modelling time dependent behaviour which has roots in statistical forecasting. There are numerous time series model structures and these are described below:

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3.1 Auto-Regressive (AR) Model

One of the simplest form of a time-series is one where the current value of a time variable is assumed to be a function of its past values only. The form of the expression is,

$$Ay(t) = \xi(t) \tag{11}$$

$\xi(t)$ is a 'chance' component used to account for uncertainty in the relationship and assumed to be identically distributed with zero mean and finite variance, σ^2. y is therefore also a random sequence. A is a *monic* polynomial (with leading coefficient 1) in the time shift operator which in time series convention is written usually as q^{-1} instead of our 'practical' interpretation of z^{-1} thus far. However, to avoid confusion, from hereon, we will not make explicit reference to the argument of the polynomials presented in equations.

Now, expansion of Eq.(11) gives:

$$y(t) = -a_1 y(t-1) - a_2 y(t-2) - \cdots - a_{N_A} y(t - N_A) + \xi(t) \tag{12}$$

Thus, the current output is a weighted sum of previous output values. Thus this type of representation is called an ‘auto-regressive’ (AR) model because its form shows a regression of a time variable with itself at different time instants. In Eq.(12), N_A defines the oldest output value that has a significant influence on the current output and the representation is termed an AR(N_A) model, or auto-regressive model of order N_A. Compare this with the difference equation, Eq. (4), where N_A defines the order of the underlying transfer function.

3.2 Moving-Average (MA) Model

Suppose we have an AR(1) model

$$y(t) = \phi y(t-1) + \xi(t) \tag{13}$$

Time shifting one unit back, we get

$$y(t-1) = \phi y(t-2) + \xi(t-1) \tag{14}$$

Substituting Eq.(14) into Eq.(13), we obtain

$$y(t) = \xi(t) + \phi\xi(t-1) + \phi^2 y(t-2) \tag{15}$$

Repeated substitution for past values of y will eventually yield an expression of the form,

$$y(t) = \xi(t) + \phi\xi(t-1) + \ldots + \phi^{t-2}\xi(2) + \phi^{t-1}\xi(1) + \phi^{t} y(0) \qquad (16)$$

Thus, in this case, $y(t)$ is re-expressed as a weighted sum (moving average) of all past ξ and its initial value $y(0)$. If the initial value is zero, then Eq.(16) can be written more compactly as

$$y(t) = C\xi(t) \qquad (17)$$

where C is a monic polynomial

$$C = 1 + c_1 z^{-1} + c_{21} z^{-2} + \ldots + c_{N_C} z^{-N_C}$$

Such a representation is known as a Moving Average (MA) time series model of order N_C, or an MA(N_C) model.

3.3 Auto-Regressive Moving Average (ARMA) Model

The AR and MA representations can be combined. For example, if we combine an AR(1) model

$$y(t) + \phi y(t-1) = \xi(t)$$

and an MA(1) model

$$y(t) = \xi(t) + \theta\xi(t-1)$$

we get $y(t) + \frac{\phi}{2} y(t-1) = \xi(t) + \frac{\theta}{2}\xi(t-1) \rightarrow \left(1 + \frac{\phi}{2} z^{-1}\right) y(t) = \left(1 + \frac{\theta}{2} z^{-1}\right)\xi(t)$

which can be expressed in the more general form

$$Ay(t) = C\xi(t) \qquad (18)$$

Equation (18) is called an Auto-Regressive Moving Average (ARMA) model. If A and C have orders N_A and N_C respectively, then Eq.(18) is an ARMA(N_A,N_C) model. ARMA models are useful in that they can provide for simpler representations when the series is an aggregate of several simpler series.

3.4 Auto-Regressive Moving Average with Exogenous Input (ARMAX) Model

The AR, MA and ARMA representations involve random sequences y and ξ.. In the control context however, actions are often taken to influence process behaviour. To include the effects of such exogenous inputs to the system in a time series model, the ARMA model can be further extended to,

$$Ay(t) = Bu(t-k) + C\xi(t) \tag{19}$$

Again, B is a polynomial in the time-shift operator and u is the manipulated or exogenous input sequence. Equation (19) describes an Auto-Regressive Moving Average with eXogenous input (ARMAX) system. Note the similarity between Eq.(19) and the difference equation of Eq.(4). However, Eq. (19) has an extra term to describe random effects. Thus the ARMAX model could be regarded as the stochastic equivalent of discrete transfer function models. Equation (19) is also called the Controlled Auto-Regressive Moving Average (CARMA) model, to state its explicit link to process control situations.

3.5 Auto-Regressive Integrated Moving Average with Exogenous Input (ARIMAX)

In all the previous time-series models, $\xi(t)$ was assumed to be an identically distributed random sequence. Passing this through the C polynomial of the moving average component enables serially correlated random effects to be modelled. In practice however, random disturbances are rarely that well behaved. Most of the times, process noise show drifting characteristics.

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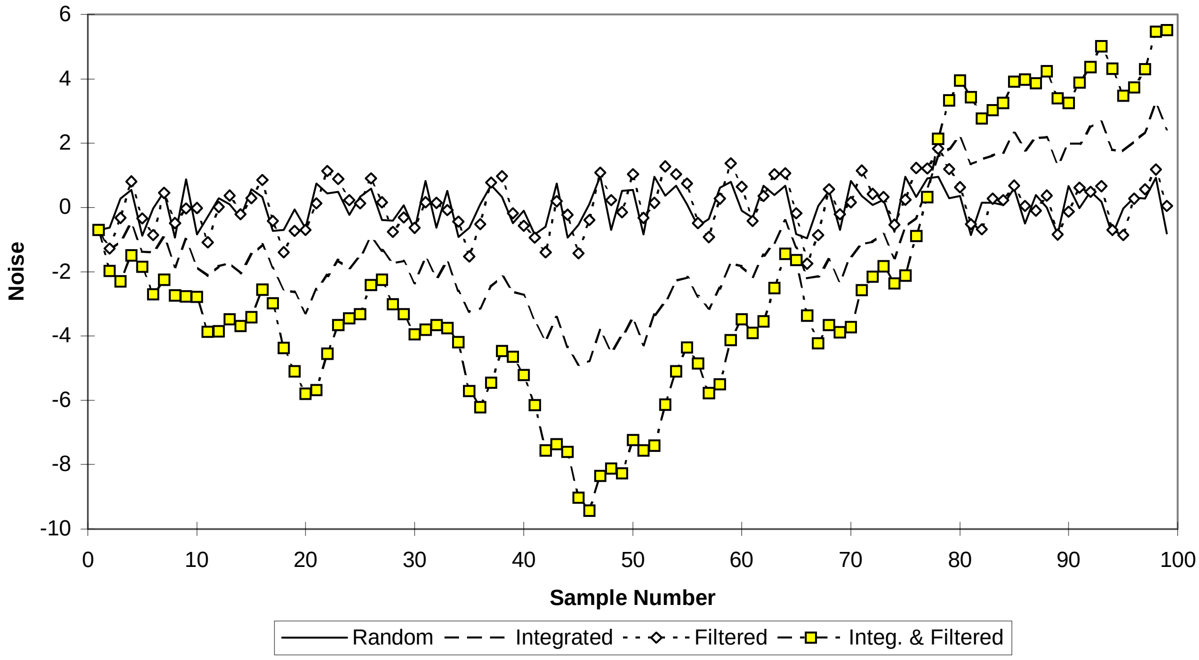


Figure 3. Different types of random noise sequences

Figure 3 shows the behaviour of 4 different types of random sequences. A normally distributed random sequence, $\xi(t)$, is shown by the continuous line. Passing it through $C = 1+0.95z^{-1}$, results in the 'filtered' or serially correlated sequence (denoted ◊). The sequences that exhibit drifting characteristics are shown by the lower two plots. These were generated respectively by the integrated random sequence, $\xi(t)/\Delta$ (dashed plot), and integrated and filtered sequence, $\frac{C\xi(t)}{\Delta}$ (denoted). C is defined as above and $\Delta = 1 - z^{-1}$. Thus to enable the modelling of drifting non-stationary behaviour, the ARMAX model can be modified to

$$Ay(t) = Bu(t-k) + C\frac{\xi(t)}{\Delta} \qquad (20)$$

which is called the Auto-Regressive Integrated Moving Average with eXogenous input (ARIMAX) time series model. A more recent terminology is to describe Eq. (20) as a Controlled Auto-Regressive Integrated Moving Average (CARIMA) model.

3.6 General Time-Series Model Representation

From the previous discussion, we note that we can construct time series from several sub-series and in a variety of configurations. In process modelling, the task then is to determine the most appropriate time-series structure and estimating the parameters of the polynomials. An even more general time series representation is given below.

$$y(t) = \frac{B}{A}u(t-k) + \frac{C\xi(t)}{D\Delta^n} \tag{21}$$

Here the contribution of u and ξ to y are clearly delineated. For a noise free system, then Eq.(21) reduces to the difference equation of Eq.(4). What is interesting is the second term of the expression. C, D and n, can be used together or in various combinations to model all practical stochastic effects.

3.7 Non-linear Time Series Models (NARMAX)

Thus far, we have only considered time series models of linear systems. Time series can also be used to model non-linear systems. For instance,

$$Ay(t) = B_1u(t-k) + B_2u(t-k)^2 + B_3u(t-k)y(t-1) + C\xi(t) \tag{22}$$

Equation (22) is an example of an NARMAX (Non-linear ARMAX) model. If appropriate, we can add more cross-product and higher order terms. Generally, if the sequence $y(t)$ is dependent on its past values, some input u and stochastic component ξ, we can write,

$$y(t) = f\left(u(t-\tau), y(t-\tau), \xi(t-\tau) | \tau \geq 1\right) + \xi(t) \tag{23}$$

where $f(.)$ is some functional. From the practical point of view, there is basically no restriction on how $f(.)$ is realised. The non-linear model could therefore be explicitly written as a NARMAX model as in Eq.(22), or the time-series represented indirectly by an Artificial Neural Network for example.

4. SUMMARY

While discrete transfer function representations have a direct correspondence to the underlying continuous system, time-series are inherently discrete in nature. In this sense, it is easy to include stochastic components into the system description as random effects are 'better' posed in the discrete domain. For example, an integrated-noise sequence in the continuous time domain has infinite variance, and theoretically, control of this process is impossible. On the other hand, the inclusion of integrated moving averages in time series does not incur this limitation. Nevertheless, transfer function models and time series models are related, and although not identical, they are often treated in the same manner when in systems analysis and controller design.

In developing a description of the process, the structure of the model is first specified. A suitable numerical technique is then used to determine the parameters of the model from input-output data gathered from the process. Transfer function and time-series models (including NARMAX structures) are linear-in-the-parameters and therefore can be parameterised by popular least-squares based algorithms. For controller design however, a '*control affine*' (linear in the manipulated input, $u(t)$) representation would be ideal. The control problem then becomes trivial. Otherwise, numerical search procedures will have to be invoked to generate an appropriate control signal.

APPENDIX 1: SAMPLED DATA SYSTEMS

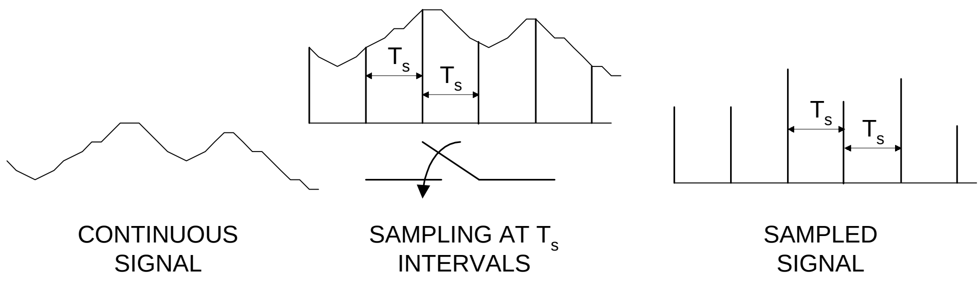
Because of the nature of digital devices, signals from plant have to be converted into a suitable form before it can be transferred for processing by a computer. Similarly, signals generated by a computer must be presented in a form suitable for receipt by the plant. The important pieces of hardware that achieve these tasks are the:

- sampler
- analog-to-digital converter (ADC)
- digital-to-analog converter (DAC)
- signal hold devices

The Sampler

The sampler is essentially a switch, operating usually at fixed intervals of time. When the 'switch' closes, it grabs or samples the output of the transmitting device. It then transfers the sampled signal to a receiver. The sampler can operate on both continuous or discrete signals.

Thus if the source signal is continuous, the output of the sampler is a series of pulses, and the magnitude of each pulse is equal to the magnitude of the continuous signal at the instant of sampling as shown in the figure below.

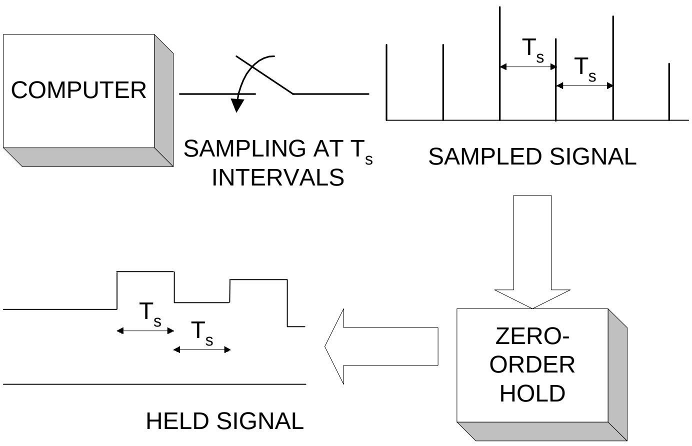


ADCs and DACs

ADCs converts sampled voltage or current signals to their binary equivalent while DACs converts binary signals to continuous signals such as voltages or currents. These converters provide the interface between a computer and the external environment.

Signal Hold Devices

The output of a sampler is a train of pulses, regardless of whether the source is continuous or discrete. Thus the output of a computer after digital-to-analog conversion is also a train of pulses. If this is a control signal, then unless the device receiving this signal, say a pump or valve, has integration capabilities, then the process will be driven by pulses. This is obviously not acceptable. So, in process control applications, the signal from the DAC is always 'held' using hardware known as signal hold devices. The most common is the Zero-Order-Hold, where each pulse is held until the next pulse comes along, that is:



Result of Zero-Order-Hold

APPENDIX 2: THE Z-TRANSFORM

The z-transform is the most commonly used tool for the analysis of sampled data systems. Suppose we sample a continuous time variable *f(t)*. Because the sampled signal exists only at the sampling instants, the sequence of pulses can be represented mathematically as:

$$f(nT_s) = \begin{cases} f(t) & \text{for } n = 0,1,2,\ldots \\ 0 & \text{otherwise} \end{cases}$$

Again, to simplify notation, let the sampled sequence be denoted by $f^*(t)$. Now the Laplace transform of *f(t)* is defined as:

$$F(s) = \int_0^\infty f(t)e^{-st}dt$$

Since the sampled signal $f^*(t)$ is a subset of *f(t)*, we can also apply the Laplace transform to it:

$$F^*(s) = \int_0^\infty f^*(t)e^{-st}dt$$

Since $f^*(t)$ only exists at sampling instants, this means that we can replace the integral with a summation, that is,

$$F^*(s) = \int_0^\infty f^*(t)e^{-st}dt = \sum_{n=0}^{\infty} f(nT_s)e^{-snT_s}$$

Defining: $z = e^{sT_s}$, then

$$F^*(s) = \sum_{n=0}^{\infty} f(nT_s)e^{-snT_s} = \sum_{n=0}^{\infty} f(nT_s)z^{-n} = F(z)$$

This is the definition of the z-transform of a continuous time signal *f(t)* sampled with a sampling interval of T_s, i.e.

$$\mathrm{Z}\{f(t)\} = F(z) = \sum_{n=0}^{\infty} f(nT_s)z^{-n}$$

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Thus, the z-transform is merely the Laplace Transform of a sampled data sequence and as such, inherits many of the properties of the Laplace Transform.

Properties

Some of the more important properties of the z-transforms are as follows:

a) Linearity

The z-transform is a linear transform. That is, given constants *a* and *b* and time variables *f(t)* and *g(t)*:

$$Z\left\{af(t)+bg(t)\right\}=aZ\left\{f(t)\right\}+bZ\left\{g(t)\right\}=aF(z)+bG(z)$$

a) z-transforms of time delays

If $f(t-kT_s)$ is *f(t)* delayed by *k* sampling intervals (*k* is an integer), and $f(t) = 0$ for $t < 0$, then the z-transform of $f(t-kT_s)$ is given by:

$$Z\left\{f(t-kT_s)\right\}=z^{-k}F(z)$$

a) Final Value Theorem

This theorem allows the calculation of the final value of a z-transformed function and is stated as:

$$\lim_{t\to\infty} f(t)=\lim_{z\to 1}(1-z^{-1})F(z)$$

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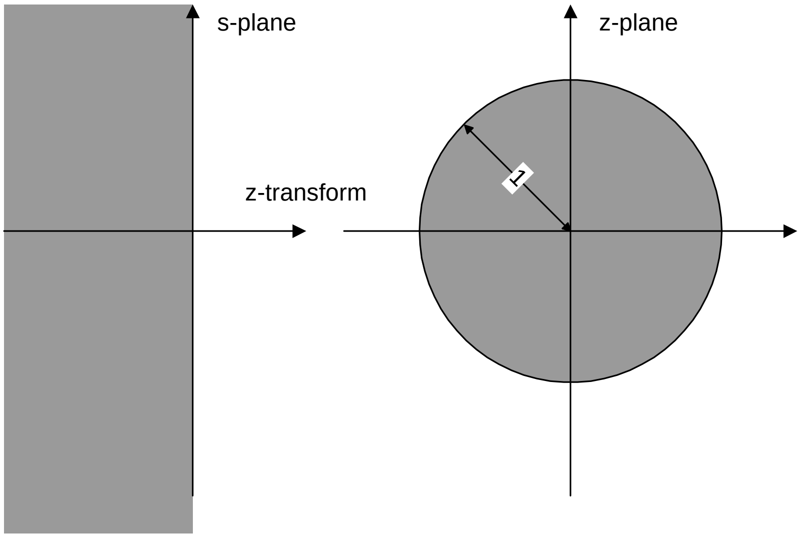
Relationship with the s-plane



The mapping from the s-plane to the z-plane is accomplished through the relationship

$$z = e^{sT_s}$$

This function maps the whole of the left side of the s-plane to a unit circle on the z-plane as shown in the above figure. In the Laplace domain, systems are stable if they do no possess poles on the right half of the s-plane. In the case of sampled data systems, they are stable if they do not possess poles that lie outside the unit-circle in the z-plane.

Example z-transforms

a) z-transform of a unit step function

The time domain representation of a unit step function is: $f(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$

Thus $Z\{f(t)\} = F(z) = \sum_{n=0}^{\infty} f(nT_s) z^{-n}$

That is, $$\begin{aligned} F(z) &= f(0) + f(T_s)z^{-1} + f(2T_s)z^{-2} + \ldots + f(nT_s)z^{-n} + \ldots \\ &= 1 + z^{-1} + z^{-2} + \ldots + z^{-n} + \ldots \end{aligned}$$

which is an infinite series, illustrating that the z-transforms operates on an infinite sequence. Fortunately, there is also a 'closed-form' equivalent, as it can be shown that

$$\frac{1}{1-z^{-1}} = 1 + z^{-1} + z^{-2} + \ldots + z^{-n} + \ldots = \frac{z}{z-1}$$

and this is the form that is always presented in z-transform tables.

b) z-transform of an exponential decay

The time function in this case is: $f(t) = \exp(-t/\tau)$

Thus $F(z) = \sum_{n=0}^{\infty} \exp(-nT_s/\tau) z^{-n} = 1 + \exp(-T_s/\tau)z^{-1} + \exp(-2T_s/\tau)z^{-2} + \ldots$

which is another infinite series. Again the closed-form solution is available and can be verified by long division to be:

$$F(z) = \frac{1}{1-\exp(-T_s/\tau)z^{-1}} = \frac{z}{z-\exp(-T_s/\tau)}$$

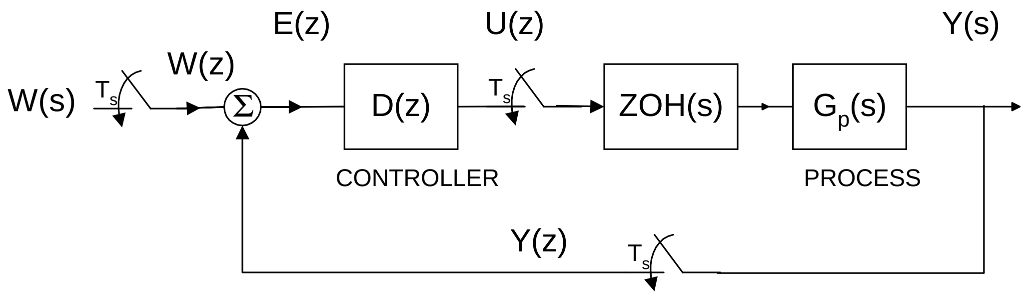
Inversion of z-transforms

Like Laplace transforms, z-transforms can be inverted back into the time domain. Given a transfer function, we can either apply long division to obtain the series form of the sampled signal or make use of the tables. The first is simple but tedious and the result may not be suitable for further analysis. Thus tables are often used. In this case, the transfer function is factored into lower order components using partial fraction expansion, and tables are used to look up the corresponding time functions of each component. The final result is obtained by adding up these individual time function.

However, due to the nature of the problem, it is not often that z-transform functions need to be converted back to the time domain. Given the range of simulation tools available nowadays, it is usually simpler to simulate the response of the discrete system to enable visualisation of response characteristics.

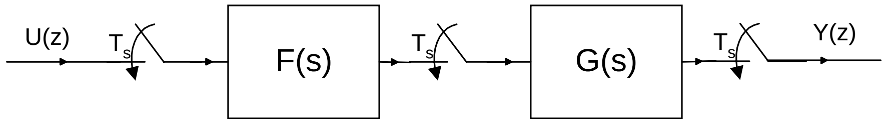
Block Diagram Manipulation

The manipulation of block diagrams of sampled data systems is very similar to that for block diagrams of systems expressed in the Laplace domain. However, because of the presence of samplers, some extra rules have to be adhered to. Consider a continuous process under digital control via the classical feedback framework. The detailed block diagram of this system including the presence of the samplers is,



Note that the samplers determine the type of signal propagating through the system. Hence the position of the samplers are important. In particular, note that the transfer function between Y(z) and U(z) for the following two systems are different.

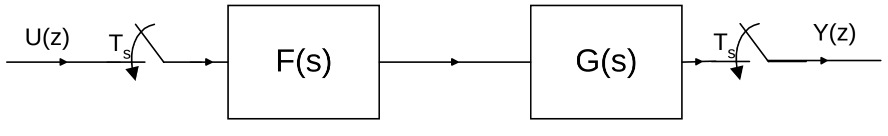
System A



Here, the relationship between Y(z) and U(z) is given by:

$$Y(z) = Z\{F(s)\}Z\{G(s)\}U(z) = F(z)G(z)U(z)$$

System B



With this system, the relationship between Y(z) and U(z) is given by:

$$Y(z) = Z\{F(s)G(s)\}U(z) = FG(z)U(z)$$

Note that in general, $Z\{F(s)\}Z\{G(s)\} \neq Z\{F(s)G(s)\}$

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